CONNECT: Volume, Surface Area

1. VOLUMES OF SOLIDS

A *solid* is a three-dimensional (3D) object, that is, it has length, width and height. One of these dimensions is sometimes called *thickness* or *depth*. It is this third dimension that distinguishes a solid from a *plane shape*. If you need to know more about plane shapes, areas and perimeters, please refer to *CONNECT: Areas, Perimeters – 1. AREAS OF PLANE SHAPES* and *CONNECT: Areas, Perimeters – 2. PERIMETERS OF PLANE SHAPES.*

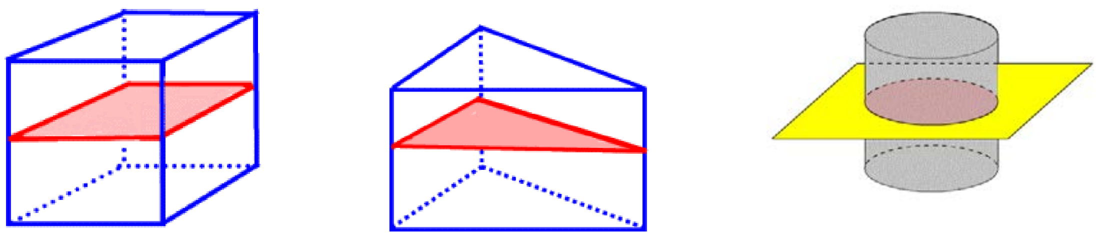
The words *volume* and *capacity* are often used interchangeably; however there is a subtle difference. "Volume is the amount of space occupied by an object or substance" whereas "capacity refers to the amount a container can hold … and generally refers to liquid measurement" (NSW Board of Studies, n. d.).

Measuring and calculating volume and capacity are done in different ways.

Volume is probably easiest to talk about if we think of a box. The boxes we are probably used to are rectangular in shape – in fact they are called *rectangular prisms*.

PRISMS

A *prism* is a solid where the cross-section is always identical and parallel to the top and bottom faces. Here are some examples. The first is a rectangular prism, the second is a triangular prism and the third is a "circular prism" otherwise known as a cylinder. The cross-sections, top and bottom faces are rectangles, triangles and circles, respectively.



Diagrams retrieved January 24, 2013, from http://www.regentsprep.org/Regents/math/geometry/GG2/PrismPage.htm and http://hotmath.com/hotmath_help/topics/cross-sections.html

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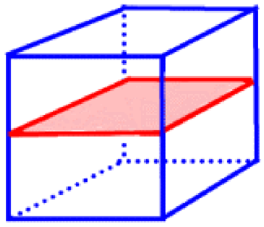
Let's talk about the rectangular prism first of all.



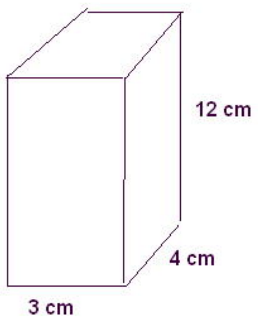
Think of the volume consisting of all the material in the container. Now try to imagine that all of that material could consist of layers of the material that have the same shape as the cross-section. Starting at the base, we would build up those layers, all the way to the top which forms the last layer. If we know the area of the cross-section (which is the same as the area of the base) and we know how high the prism is, then we can multiply the area of the base by the height and the result is the volume.

In the rectangular prism, the cross-section is a rectangle. We know that the area of a rectangle is $A = l \times w$ units^2, (where A units^2 is the area, l units is the length of the rectangle and w units is the width of the rectangle). If we let h units be the height of the rectangular prism and V units^3 be its volume, then the volume is given by

$$V = l \times w \times h \text{ units}^3$$

We use cubic units (units^3) because there are 3 dimensions to take into account with volume. The units we use could be cm^3, mm^3, m^3 and so on.

Example: Calculate the volume of this rectangular container:



$V = l \times w \times h$

In this case, l = 3cm, w = 4cm, h = 12cm

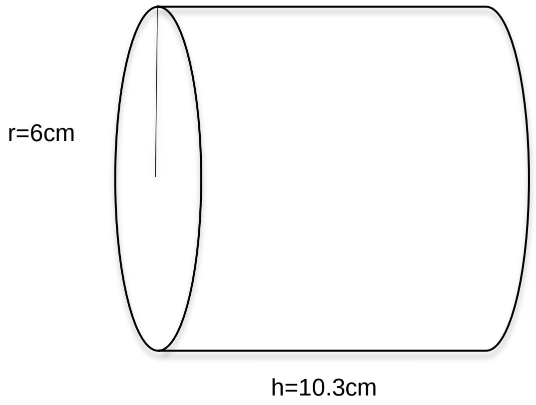
so V = 3cm x 4cm x 12cm

=144 cm^3

That is, the volume is 144 cm^3.

The volumes of other prisms are calculated in the same way. Find the area of the base (or cross-section) and multiply that by the height.

Note that the diagram of the prism might show it lying on its side. Example: calculate the volume of this prism (cylinder):



The “base” of this cylinder is a circle because if you cut a cross-section vertically through the cylinder, you would get a circle. So the volume is found by calculating the area of the circle and multiplying by the height.

For the circle, Area = πr^2

$= \pi \times 6^2$ cm^2

= 113.0973355… cm^2

Keeping that result in the calculator (that is, without rounding), we then multiply by h.

So the volume is 113.0973355… cm^2 x 10.3cm = 1164.902556…cm^3.

Round that answer to find the volume, it is (approximately) 1164.903cm^3.

Over the page is one for you to try. You can check your result with the solution at the end of the resource.

Calculate the volume of the figure.

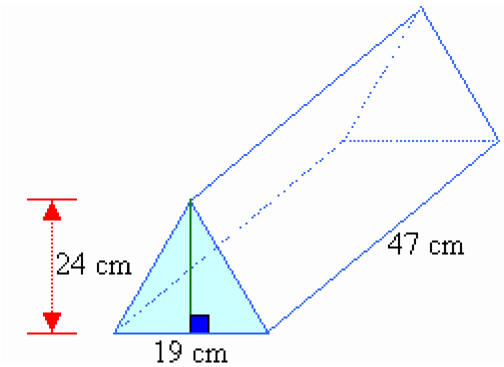


Diagram retrieved January 24, 2013, from
http://www.mathsteacher.com.au/year8/ch13_volume/02_prisms/prisms.htm

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Converting between cubic units

We need to remember that we are dealing with 3 dimensions when converting between cubic units.

For example, this diagram represents a centimetre cube where all the dimensions are 1cm in length.

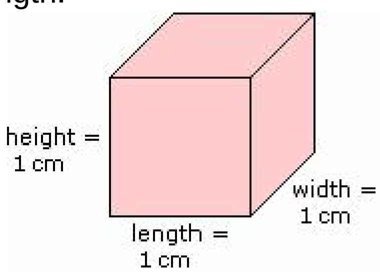


Diagram retrieved January 24, 2013, from
http://betaus.icoachmath.com/math_dictionary/Cubic_Centimeter.html

When we find the volume of the cube, we get

$$V = 1cm \times 1cm \times 1cm$$

$$= 1cm^3$$

But we can also measure each dimension as 10mm, (1cm = 10mm) and using this measure to calculate the volume, we get

$$V = 10mm \times 10mm \times 10mm$$

$$= 1000mm^3$$

That means

$$\boxed{1cm^3 = 1000mm^3}$$

In the same way, $1m^3 = 100cm \times 100cm \times 100cm$

$$= 1000000cm^3$$

Converting between volume *units and* capacity *units*

We have seen above that volume is measured in cubic units such as metres^3, centimetres^3 and so on.

Capacity is measured in litres (L), millilitres (mL), kilolitres (kL) and so on.

It can be shown that one cubic centimetre holds one millilitre of water, that is,

$$\boxed{1cm^3 \text{ holds } 1mL}$$

and one cubic metre holds 1000 litres, that is

$$\boxed{1m^3 \text{ holds } 1000L}$$

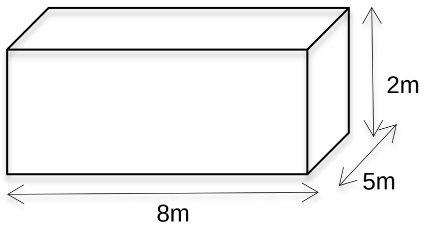
We also know that 1000L is 1kL, so

$$\boxed{1m^3 \text{ holds } 1kL}$$

We could calculate the amount of liquid a solid might hold by converting between units.

For example, a swimming pool is rectangular in shape and has length 8m, width 5m and can be filled to a height of 2m. How many litres of water could it hold?

First, we draw the pool (not to scale):



Volume of the pool = 8m x 5m x 2m

= $80m^3$

Each m^3 holds 1000L, and so the pool holds 80 x 1000L, that is 80000L.

(We can also do this in kL: each m^3 holds 1kL, so the pool can hold 80kL.)

Here is one for you to try. You can check your result with the solution at the end of the resource.

Calculate the volume of a cylinder that has a radius of 10cm and height of 20cm. How many litres can the cylinder hold? (It's a really good idea to draw a diagram to help you complete this sort of problem.)

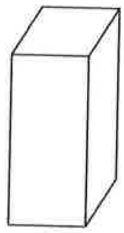
PYRAMIDS

Another kind of solid you might be interested in is a pyramid.

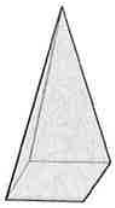
Retrieved March 25, 2013, from http://www.flickr.com/photos/scottsm/8399190381/in/photostream

It can be shown that the volume of a pyramid is one third the volume of its corresponding prism. "Corresponding" means that the pyramid and prism both have the same sized base and height, but the pyramid's side faces meet at a point, called the vertex. We can have a rectangular pyramid (the corresponding prism is a rectangular prism), a square pyramid (corresponding prism is a square prism), a triangular pyramid (corresponding prism is a triangular prism), a circular pyramid, which we know as a *cone* (corresponding prism is a cylinder), a hexagonal pyramid …

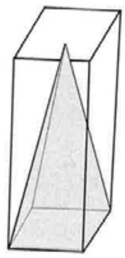
Other than the cone, all pyramids' side faces are triangles. Here are examples of a square pyramid and corresponding square prism:



Square prism



Square pyramid



Square pyramid inside square prism

Diagrams retrieved January 30, 2013, from http://www.ck12.org/book/CK-12-Foundation-and-Leadership-Public-Schools%252C-College-Access-Reader%253A-Geometry/r1/section/7.6/

When calculating the volume of a pyramid, we need to know the *height* of the pyramid, just as we need to know it to calculate the volume of a prism. The height that we need to know MUST be the *perpendicular* height of the pyramid, that is, it must be at right angles with the base of the pyramid.

Example. We need to calculate the volume of a rectangular pyramid, with a base length 3m, width 2m and perpendicular height 2.5m.

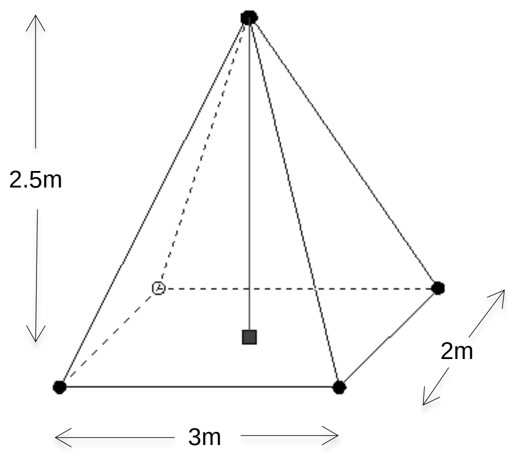


Diagram retrieved January 30, 2013, from http://www.mathe-schumann.de/tokyo/schumannlectureICME9.htm

The corresponding prism would be rectangular, its volume would be the area of the rectangular base times the height.

Area of the base = 3m x 2m

= $6m^2$

Volume of prism = $6m^2$ x 2.5m

=$15m^3$

So, the volume of the pyramid is $\frac{1}{3} \times 15\text{m}^3$ = $5m^3$

Here is one for you to try. You can check your result with the solution at the end.

Calculate the volume of a cone where the radius of the circular base is 10.3cm and its height is 20cm.

How many mL does the cone hold? How many litres is this?

SPHERES

Spheres are special because they do not fit the prisms or pyramids categories. If you need to calculate the volume of a sphere, there is a special formula:

$V = \frac{4}{3}\pi r^3$ units^3

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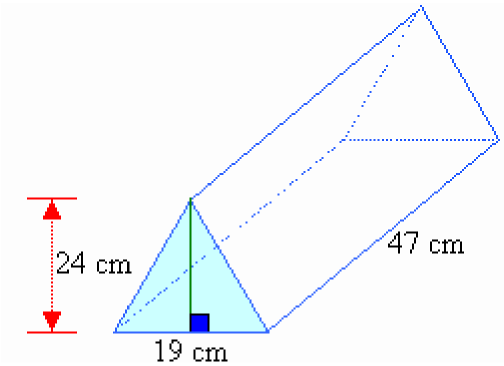
If you need help with any of the Maths covered in this resource (or any other Maths topics), you can make an appointment with Learning Development through Reception: phone (02) 4221 3977, or Level 3 (top floor), Building 11, or through your campus.

Solutions

Volume of prism (page 4)



The "base" (or cross-section) of this prism is a triangle. The area of a triangle is A = ½ x b x h units ^{2}, where A units^2 represents the area of the triangle, b units represents the length of the base of the triangle and h units represents the height of the triangle.

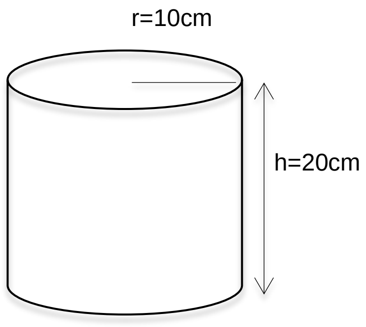
Here the length of the base (b) of the triangle is 19cm, and the height (h) of the triangle is 24cm, so the area of the triangle is

$$A = \tfrac{1}{2} \times 19 \times 24 \text{cm}^2$$

$$= 228 \text{ cm}^2$$

So, the volume of the prism is 228cm^2 x the height of the prism, that is 228cm^2 x 47cm, which is 10716cm^3.

Volume of cylinder (page 6)



Area of circle (cross-section) = πr^2

$= \pi \times 10^2$ cm^2

= 314.1592654… cm^2

Volume of the cylinder = 314.1592654… cm^2 x 20cm

= 6283.185307…cm^3

Each cm^3 of water holds 1 mL, so the cylinder holds 6283.185307…mL. To find this in litres, divide by 1000, and obtain 6.283185307…L. Now round that answer to 6.283L, so the cylinder holds approximately 6.283L.

Volume of cone (page 8)

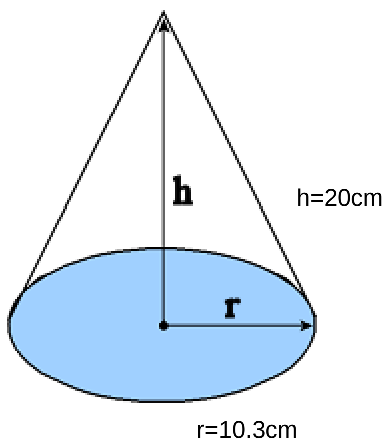


Diagram retrieved January 30, 2013, from *http://hawaii.hawaii.edu/math/Courses/Math100/Chapter2/Text/Text2207.htm*

Corresponding prism is a cylinder, radius 10.3cm, height 20cm.

Volume of cylinder = $\pi r^2 \times h$

$= \pi \times 10.3^2 \times 20$ cm^2

= 6665.831292… cm^2

So, volume of cone = $\frac{1}{3} \times 6665.831292\ldots$cm^3

= 2221.943764…cm^3

That is, the volume of the cone is approximately 2221.943cm^3.

REFERENCES

NSW Board of Studies (n. d.). *NSW Syllabuses for the Australian Curriculum. Mathematics K-10 – Stage 2 – Measurement and Geometry: Volume and Capacity.* Retrieved February 18, 2013, from http://syllabus.bos.nsw.edu.au/mathematics/mathematics-k10/content/1127/