MATH141 – Autumn 2008
Tutorial Sheet – Week 12

Solutions available as of Friday at the MATH141 web site:
`http://www.uow.edu.au/~mnelson/teaching.dir/math141.html`

1. Let $\underset{\sim}{a} = (1,0,1)$, $\underset{\sim}{b} = (1,2,1)$ and $\underset{\sim}{c} = (1,1,0)$. Evaluate the following.

   (a) $\underset{\sim}{a} \times \underset{\sim}{c}$ (b) $\left[\underset{\sim}{b}, \underset{\sim}{a}, \underset{\sim}{c}\right]$ (Are $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ coplanar?) (c) $\left(\underset{\sim}{b} \times \underset{\sim}{a}\right) \times \underset{\sim}{c}$ (d) $\underset{\sim}{b} \times \left(\underset{\sim}{a} \times \underset{\sim}{c}\right)$.

2. Express the polar equation $r^3 = \dfrac{4}{2\cos\theta - 3\sin\theta}$ in Cartesian coordinates.

3. Use the scalar triple product to varify that the vectors $\underset{\sim}{a} = (2,3,1)$, $\underset{\sim}{b} = (1,-1,0)$ and $\underset{\sim}{c} = (7,3,2)$ are coplanar.

4. (a) Find the antiderivatives of $\sin x$ and x^2.

   (b) Evaluate the definite integrals $\displaystyle\int_{\pi}^{0} \sin x \, \mathrm{d}x$ and $\displaystyle\int_{0}^{3} x^2 \, \mathrm{d}x$.

5. Find the distance between the skew lines

   $$\mathcal{L}_1 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -4 \\ 5 \end{pmatrix} s \text{ and } \mathcal{L}_2 : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} 8 \\ -3 \\ 1 \end{pmatrix} t.$$

6. Convert the Cartesian coordinate equation $x^2 - 6x + y^2 = 0$ to a polar coordinate equation.

7. Let the 3 points, U, V and W (in 3 dimensions) have position vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$, $\underset{\sim}{w}$ respectively.

   (a) Find $\overrightarrow{UW}$ and $\overrightarrow{UV}$.

   (b) Prove that if U, V and W are colinear (That is, lie on the same line), then

   $$\underset{\sim}{u} \times \underset{\sim}{v} + \underset{\sim}{v} \times \underset{\sim}{w} + \underset{\sim}{w} \times \underset{\sim}{u} = \underset{\sim}{0}.$$

8. Find the following derivatives

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{2}^{x} \left(3t^3 - 4t^2 + 3t - 1\right) \mathrm{d}t$ (b) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{x}^{3} \left(t - \ln t\right) \mathrm{d}t$ (c) $\dfrac{\mathrm{d}}{\mathrm{d}x} \displaystyle\int_{0}^{\sin x} \sqrt{1 - t^2} \, \mathrm{d}t$.

9. Find the value(s) of r when $\theta = \dfrac{6.5\pi}{12}$ if $r = 5\cos 3\theta$.

10. Using the polar graph paper provided in your text book sketch the function $r = 5\cos 3\theta$, $0 \le \theta \le 2\pi$. The following set of values may be useful

θ	$\frac{0\pi}{12}$	$\frac{1\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12}$	$\frac{7\pi}{12}$	$\frac{8\pi}{12}$	$\frac{9\pi}{12}$	$\frac{10\pi}{12}$	$\frac{11\pi}{12}$	$\frac{12\pi}{12}$	$\frac{13\pi}{12}$
r	5	3.5	0	-3.5	-5	-3.5	0	3.5	5	3.5	0	-3.5	-5	-3.5
θ	$\frac{14\pi}{12}$	$\frac{15\pi}{12}$	$\frac{16\pi}{12}$	$\frac{17\pi}{12}$	$\frac{18\pi}{12}$	$\frac{19\pi}{12}$	$\frac{20\pi}{12}$	$\frac{21\pi}{12}$	$\frac{22\pi}{12}$	$\frac{23\pi}{12}$	$\frac{24\pi}{12}$			
r	0	3.5	5	3.5	0	-3.5	-5	-3.5	0	3.5	5			

Week 12 Lecture Material

FUNDAMENTALS	**ALGEBRA**
(Mark Nelson)	(Nirmalendu Chaudhuri)
Sections 3.1, 3.2, 3.3.1–3.3.4	Sections 6.14.3, & 6.14.4
Exercises 3.2.3, 3.3.2.1, 3.3.3.1, 3.3.4	**Exercises 6.15**