MATH141 – Autumn 2008
Tutorial Sheet – Week 9

Solutions available as of Friday at the MATH141 web site:
`http://www.uow.edu.au/~mnelson/teaching.dir/math141.html`

1. Let $\underset{\sim}{u} = \underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{v} = \underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{w} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$. Find

   (a) the magnitude of $\underset{\sim}{u} + \underset{\sim}{v} - \frac{1}{2}\underset{\sim}{w}$ (i.e., find $\left|\underset{\sim}{u} + \underset{\sim}{v} - \frac{1}{2}\underset{\sim}{w}\right|$),

   (b) $u + v - \frac{1}{2}w$ (note: $u = \left|\underset{\sim}{u}\right|$, etc),

   (c) $\dfrac{\underset{\sim}{w}}{w}$,

   (d) the magnitude of $\dfrac{\underset{\sim}{w}}{w}$.

2. Find inverses of the following functions.

   (a) $x^2 - 2x, \quad x \geq 1$ (b) $\dfrac{4x}{x-2}$

3. Let $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j}$, and $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j}$. Represent these diagramatically as position vectors, and plot the points with position vectors $\underset{\sim}{a} + \underset{\sim}{b}$ and $\frac{1}{2}\left(\underset{\sim}{a} - \underset{\sim}{b}\right)$.

4. (a) Simplify $\sin\left(\cos^{-1} x\right)$, for $|x| < 1$.

   (b) Prove that $\cosh^2 x - \sinh^2 x = 1$.

   (c) Prove that $\tanh^{-1} x = \frac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, for $|x| < 1$.

5. $ABCD$ is the parrallelogram shown in figure 1.

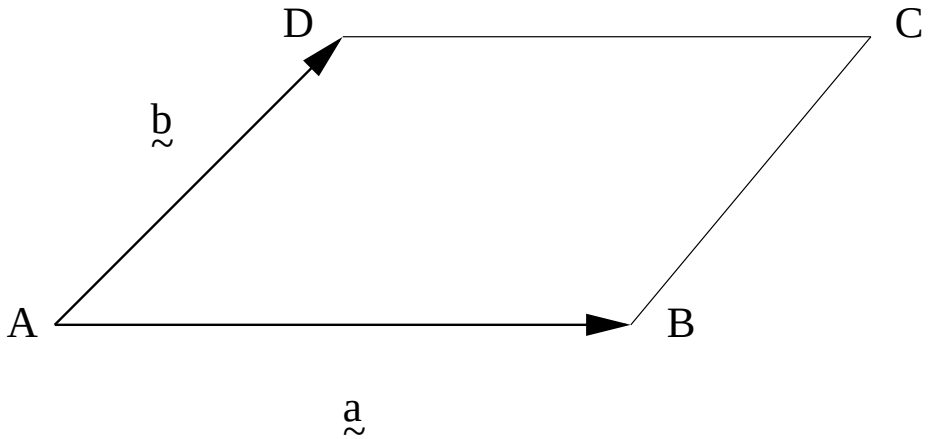

Figure 1:

   Find $\overrightarrow{BD}$ and $\overrightarrow{CE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, where E is the midpoint of AD.

6. (a) Simplify $\cos(\arcsin x)$ for $|x| < 1$.

   (b) Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$.

   (c) Prove that $\coth^{-1} x = \frac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right)$, for $|x| > 1$.

7. The rectangle in figure 2 is such that $ABEF$ is a square and $BC = 2AB$. If $\overrightarrow{AF} = \underset{\sim}{a}$, and $\overrightarrow{AB} = \underset{\sim}{b}$, find $\overrightarrow{BC}$, $\overrightarrow{BD}$, $\overrightarrow{FC}$, $\overrightarrow{AD}$ and $\overrightarrow{EC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

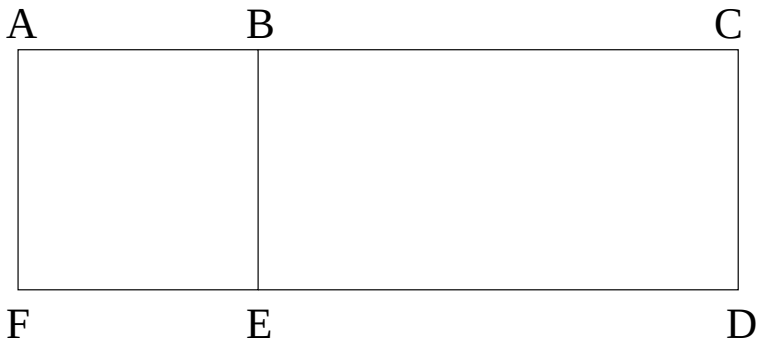

Figure 2:

8. **How large is $n!$?**

   $n!$ is the number of ways in which it is possible to arrange n objects. But how big is $n!$?

   Imagine that you have a new digital camera and you want to photograph a group of people in a row, and you want to do it in every way they can be lined up (and left-to-right looks different from right-to-left). Suppose that there are two people in the photograph: Alice and Bob. Then there are $2! = 2$ ways to arrange them:

   Alice-Bob or Bob-Alice.

   If there are three people (Alice, Bob & Chris) then there are $3! = 6$ ways to arrange them:

   Alice-Bob-Chris, Alice-Chris-Bob, Bob-Alice-Chris, Bob-Chris-Alice, Chris-Alice-Bob, Chris-Bob-Alice.

   Let's suppose that it takes fifteen seconds to go from one arrangement to another (this assumes we have a systematic way of doing this). Then it takes $2! \times 15 = 30$ seconds to take all the photographs of Alice and Bob and $3! \times 15 = 90$, seconds to take all the photographs of Alice, Bob and Charlotte.

   How long does it take to take all of the photographs if there are:

   (a) 4 people. (convert your answer to minutes)
   (b) 8 people. (convert your answer to days)
   (c) 12 people. (convert your answer to years)

   L.J. Lipkin. 2006. *How large is n!*. The College Mathematics Journal **37**(2): 109.

9. Let $\underset{\sim}{u} = \underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{v} = \underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{w} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$. Find

   (a) the magnitude of $\underset{\sim}{u} + \underset{\sim}{v}$ (i.e., $\left|\underset{\sim}{u} + \underset{\sim}{v}\right|$,
   (b) $u + v$,
   (c) $w\underset{\sim}{w}$,
   (d) the magnitude of $w\underset{\sim}{w}$ (i.e., $\left|w\underset{\sim}{w}\right|$).

Week 9 Lecture Material

FUNDAMENTALS	**ALGEBRA**
(Mark Nelson)	(Nirmalendu Chaudhuri)
Sections 2.7, 2.8, 2.9	Sections 6.8 & 6.10
Exercises 2.8.1, 2.9.1	**Exercises 6.6 & 6.9**