

Implicit Differentiation Worksheet: Solutions

1. (a)

$$\begin{aligned}\sin y &= x^2 \\ \cos y \frac{dy}{dx} &= 2x \\ \frac{dy}{dx} &= \frac{2x}{\cos y}\end{aligned}$$

(b)

$$\begin{aligned}x \cos y + y &= x^3 \\ \frac{d}{dx}(x \cos y) + \frac{d}{dx}(y) &= \frac{d}{dx}(x^3) \\ \cos y \frac{d}{dx}(x) + x \frac{d}{dx}(\cos y) + \frac{d}{dy}y \cdot \frac{dy}{dx} &= 3x^2 \\ \cos y \cdot 1 + x \frac{d}{dy}(\cos y) \cdot \frac{dy}{dx} + 1 \cdot \frac{dy}{dx} &= 3x^2 \\ -\sin y \cdot x \frac{dy}{dy} + \frac{dy}{dx} &= 3x^2 - \cos y \\ (1 - x \sin y) \frac{dy}{dx} &= 3x^2 - \cos y \\ \frac{dy}{dx} &= \frac{3x^2 - \cos y}{1 - x \sin y}\end{aligned}$$

(c)
$$\frac{dy}{dx} = \frac{\cos(x-1)}{2y}$$

(d)
$$\frac{dy}{dx} = \frac{x}{2 \sin(2y) \sqrt{1-x^2}} = \frac{1}{2} (1-x^2)^{-1/2}$$

(e)
$$\frac{dy}{dx} = e^x (x+1) y = e^x (x+1) \exp(xe^x)$$

(f)
$$\frac{dy}{dx} = \frac{3e^{3x}}{e^y} = \frac{3e^{3x}}{5 + e^{3x}}$$

(g)
$$\frac{dy}{dx} = \frac{2x}{1 + 3y^2}$$

(h)
$$\frac{dy}{dx} = \frac{\cos x}{2y + \cos y}$$

(i)
$$\frac{dy}{dx} = \frac{1-y}{x+1-2y}$$

2.
$$\frac{dy}{dx} = \frac{2x}{e^{\sin y} \cos y + 1}$$

3.
$$\frac{dy}{dx} = \frac{e^{\sin y}}{e^y - x \cos y e^{\sin y}}$$

$$4. \frac{dy}{dx} = \frac{-2xy^3 - 2x^2y - 1}{x^2y^2 + x^3 + 2y}$$