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OPTIMAL GENERATION PLANNING FOR ELECTRICITY GRIDS  
CONTAINING WIND FARMS

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ABSTRACT

Numerical probabilistic models have been used to re-optimize the mix of thermal base, intermediate and peak load plant in electricity grids containing specified wind power capacity. Wind power plant displaces thermal baseload capacity equal to slightly more than the average wind power output. To maintain grid reliability (LOLP), additional thermal peaking plant, rated at about half the average wind power, has to be installed. The breakeven costs of wind power in a model British CEBG, containing coal, nuclear, oil and wind driven power plant, are evaluated under a wide range of conditions. Wind farms, based on existing commercial aerogenerators may already be economically competitive in this grid. Spot pricing of electricity may offer an efficient way of matching wind power to load.

KEYWORDS

Wind power; optimal mix; generation expansion; generation planning;  
systems integration.

INTRODUCTION

This paper surveys recent work, both published and as yet unpublished, by a Canberra based wind energy research group, on the economic value of wind farms in electricity grids.

Early approaches to the economics of wind farms tended to calculate their costs and then to assume incorrectly that their economic values were only those of the fuel saved in thermal power stations or water saved in hydro-electric storages. It was assumed, explicitly or implicitly, that conventional power stations are "firm" or reliable sources of power, while wind driven generators are unreliable, unless they have storage.

This assumption was a value judgement rather than a scientific or technical one. An empirical approach demonstrates that all power plant is unreliable to some extent. Forced outages of thermal power plant, which are result of

breakdowns, tend to occur less frequently than forced outages of wind farms, which mainly result from lulls in the wind. But lulls tend to have shorter durations. Therefore, a quantitative analysis is required, starting from widely acceptable definitions of reliability of individual power plants and of whole generating systems.

But, although quantitative, the formulation cannot be deterministic, because it must reflect the uncertainties inherent in the problem: in the load  $L$ , the power  $A$  available from conventional power stations and the wind power  $W$ . This is done by treating  $L$ ,  $A$  and  $W$  as random variables, each of which has an empirical probability distribution function.

#### OPTIMAL MIX OF A 3-COMPONENT THERMAL GRID WITH A WIND FARM

Consider first an electricity grid with three types of thermal power station labelled "base", "intermediate" and "peak". In general, the fixed annual cost of base plant is much greater than that of peak, but the annual variable cost of peak is much greater than that of base. Therefore, there is an optimal mix of base, intermediate and peak plant which minimizes the total annual cost of the generating system.

This optimal mix has been illustrated in some textbooks (e.g. Marsh, 1980) by means of a "cost polygon" obtained by plotting cost against operating time for each type of plant given its characteristic capital and running costs. In its basic form this method makes no allowance for forced outages which require a probabilistic treatment (e.g. Gates, 1983).

In the numerical probabilistic models developed by the Canberra group (Martin and Diesendorf, 1982, 1983), each of the types of thermal unit is characterised by its annualised fixed and variable costs in constant currency, economic lifetime, rated power, minimum power, forced outage rate and planned outage rate. Within a particular type, individual power units are treated as identical. The probability distribution function of power available from the thermal units is derived from the respective outage rates of the individual types of plant. Wind power is described by a Rayleigh distribution function of wind speeds which has been filtered by the response (linear between cut-in and rated wind speeds) of a standard aerogenerator. The probability distribution of load is determined by the "load duration function" (Appendix).

The planning program chooses units from three different types of new thermal power plant to construct a generating system with optimal mix to meet the empirical load duration curve. The annual economic cost of a mix of units is the sum of annualised capital costs of each unit, plus the sum of fuel "cycle" costs per unit of energy generated times the energy generated by that unit. The optimal mix is then the mix of the three types of thermal unit which minimizes total annualised cost, given a grid reliability specified by the Loss of Load Probability (LOLP), the fraction of hours per year for which available power fails to meet load (see Appendix).

The economic value of wind power is then obtained by re-optimising the thermal mix in the presence of a specified wind power capacity, given that LOLP is kept the same. Thus the capacity credit of wind power (Martin and Diesendorf, 1980; Haslett and Diesendorf, 1981), is taken into account automatically in this formulation. That is, we permit wind farms to substitute for thermal power plant in addition to the fuel burnt in such plant. In the presence of wind farms, an "effective load distribution" is derived by "subtracting" the wind power from the load. (The "subtraction" is actually a numerical convolution

of the probability distributions of load and wind power. Only in the particular case when there are simultaneous data sets of load and wind power can one perform an ordinary subtraction of data points.)

The results for the 3-component (+ wind farm) grid (Martin and Diesendorf, 1983) confirm and extend earlier work by the Canberra group (Martin and Diesendorf, 1982; Diesendorf, Martin and Carlin, 1981), who calculated the breakeven cost of wind power in the special case of a 2-component thermal grid. In the 3-component thermal grid, minimum annual cost is still achieved when a wind farm substitutes for somewhat more than its average power in thermal base load units. To maintain grid reliability (LOLP), additional thermal peak load plant, equivalent in rating to about half the average wind power, has to be installed. But since this peak plant is rarely used, and has low capital cost, it plays the role of a reliability insurance with a low premium.

In the particular case when the 1980 cost parameters (CEGB, 1980) of new nuclear, coal and oil fired units of the British CEBG grid are assigned to the thermal base, intermediate and peak load units respectively, and the observed load duration curve of that grid is utilized, the breakeven cost of 1 GW of mean wind power, equivalent to about 4% annual grid energy generation, is £572 per rated kilowatt (Martin and Diesendorf, 1983). For this result, the annual mean wind speed at 10m height was taken to be  $6 \text{ ms}^{-1}$ , the real annual interest rate 5%, economic lifetime 30 years, and annual operation and maintenance 2% of capital cost.

Since a single commercial Danish 55 kw aerogenerator was imported into Australia and connected to the Victorian grid at Ballarat in February 1983 for about 1980 A\$1,000/rated kw total cost, it is likely that a wind farm, for which site costs are spread over many similar aerogenerators, could be installed in Britain for less than 1980 £572/rated kw (assuming that £1 = A\$2) and so that wind power may already be economical in the CEBG grid.

The manipulation of the probability distribution functions of load, availability and wind power is computationally fast and so a sensitivity analysis of the results to changes in economic, grid, power station and wind parameters is readily performed (Martin and Diesendorf, 1983).

In determining the optimal thermal mix for a variety of economic and other parameters, we have found that this mix almost never consists of a configuration of all three types of unit. For the case of the baseline model of the 1980 CEBG grid, the optimal mix in the absence of wind power is coal-oil, in agreement with simple cost of energy calculations (Jeffery, 1980, 1982). Nuclear power at £1085/kw (CEBG, 1980) is not competitive in this case.

Breakeven costs of wind power. A 50% increase in the real cost of coal fuel raises the breakeven cost of wind power (in £/rated kw) to 631, thus making wind power even more competitive economically, while a 50% increase in the cost of oil fuel (all other parameters remaining at their baseline values) lowers the breakeven cost to 460. Doubling the penetration of wind power capacity gives 522, while increasing or reducing the annual mean wind speed to  $6.5 \text{ ms}^{-1}$  or  $5.5 \text{ ms}^{-1}$  gives 662 or 469 respectively. Increasing all capital costs by 50% gives 631 and increasing all fuel costs by 50% gives 717. Doubling the real interest rate gives 439.

Gates (1983) has obtained exact analytic solutions for the static optimal mix of an electricity grid consisting of n types of thermal power unit. In the Appendix, Gates' models are outlined and the method for extending them to

incorporate wind power is indicated.

#### NON-OPTIMAL MIX

If no attempt is made to reoptimize the thermal mix of an electricity grid when wind power is introduced, then the economic value of the capacity credit of wind power will be in general less than that obtained in the previous section, and will require separate calculation.

The Canberra group evaluated the capacity credit of wind power by means of a numerical probabilistic model (Martin and Diesendorf, 1980) and an analytic probabilistic model (Haslett and Diesendorf, 1981). Carlin (1983) included wind-load correlations in the analytic model, while Martin and Carlin (1983) included them in the numerical model. Haslett (1981) incorporated the effect of spatial correlations between dispersed sites into the analytic model. The analytic papers drew upon various statistical models of the probability distribution function of wind power output from aerogenerators at a single site or multiple sites, developed by Haslett and Diesendorf (1981), Haslett and Carlin (1981) and Carlin and Haslett (1982).

Of the non-Australian papers on calculating the capacity credit of wind power, Janssen (1982), which contains an extensive reference list, is of particular interest because it takes into account explicitly the effects of several constraints on the operation of thermal power plants.

#### SPOT PRICING OF ELECTRICITY

Consider a future electricity grid containing a large component of generating plant in the form of wind farms. This grid has incorporated the advances in microelectronics and power electronics which are already available in the early 1980s, and also a system of tariffs (experimental in one or two grids in 1983) called *spot pricing*.

During periods of high wind and low load, an ordinary 1983 grid would have to spill excess wind energy. However, in our future grid, electricity consumers have switches on their hot water circuits which are sensitive to small changes in grid frequency. Before leaving for work in the morning, most of the consumers have programmed their personal microcomputers with the maximum prices they are prepared to pay that day for electricity on the hot-water circuit, refrigerator circuit, etc. When load drops, the spot price of electricity drops too, and this information is transmitted over the mains to the customers' microcomputers. Where a strong wind begins to surge across the wind farms, the grid frequency increases slightly, the spot price of electricity drops even further, and electricity is switched automatically into millions of hot water boosters.

Under spot pricing (Schweppe and coworkers, 1980; Outhred and Schweppe, 1980; Caramanis, Bohn and Schweppe, 1982; Caramanis, 1982), the value of electricity varies with time, depending on supply and demand factors. A breakdown in a thermal power unit, a lull in the wind or an increase in load lifts the spot price of electricity; conversely, maximum availability, good winds or low loads reduce the spot price. In other words, the grid becomes a "market place", in which each of the participants, whether a supplier or a consumer, seeks to maximize his/her individual profits. In this market the price of electricity is not predetermined (as it is in the case of time-of-day pricing, for example), but rather is the result of optimizing a social welfare function

for the supply/demand system. Optimization is carried out by adjusting the generation/consumption pattern of each participant, subject to the usual engineering constraints.

With spot pricing, there are no customer classes, no demand charges, no capacity credit and no distinction between buying and selling at the margin. Electricity supplied at a given point in the network at a given time has the same value, whether it is supplied by a coal-fired station, a wind farm or a portable diesel generator. This offers a greater flexibility in supply and permits new or dispersed sources of electricity to compete free of the traditional handicaps.

The introduction of spot pricing would not be justified solely on the grounds of giving a fair deal to dispersed, renewable sources of electricity such as wind power. Under spot pricing, customers would receive clear signals about wasteful uses of electricity (Bannister, 1983) and utilities would receive disincentives against inappropriate investments. Furthermore, the arguments about appropriate buy-back rates would be avoided.

A first step towards such a more efficient tariff system has been the introduction of *time-of-day tariffs* by a few utilities. For example, the Pacific Gas and Electric Company of California pays cogenerators and small power producers, including the owners of wind farms, a tariff which varies from 6 c/kWh during off-peak periods to about 13 c/kWh during peak periods (PG&E, 1983). However, time-of-day tariffs can only resolve a small part of the problems of economic inefficiency and equity in current electricity pricing systems.

#### APPENDIX: EXACTLY SOLUBLE MODELS

We outline here the basic mathematical approach for deriving exact analytic formulae for the static optimal mix of electricity grids with  $n$  types of thermal power station (Gates, 1983). Let there be  $N_i$  power plants of type  $i$ , with total capacity  $C_i$  and total available power  $A_i$  at time  $t$ .  $A_i$  is a random variable. Then if  $N_i$  is sufficiently large, the probability distribution function of  $A_i$  is Normal. The probability distribution function of the load  $L$  can be modelled quite well for large grids by the shifted Rayleigh distribution

$$F_L(l) \equiv \Pr(L \leq l) = \begin{cases} 0 & \text{if } 0 \leq l \leq l_0 \\ 1 - \exp\left[-\pi(l-l_0)^2/4(\bar{l}-l_0)^2\right] & \text{if } l > l_0 \end{cases} \quad (\text{A.1})$$

where  $l_0$  and  $\bar{l}$  are parameters which are determined by fitting  $1 - F_L(l)$ , obtained from equation (A.1), to the empirical *load duration curve*. One possible particular choice of parameters is  $l_0 = \text{minimum load}$ ,  $\bar{l} = \text{mean load}$ .

Optimal mix is achieved by minimizing a cost function subject to the constraint that LOLP is equal to a constant value  $p$  specified by utility policy:

$$\left\langle \Pr\left\{\sum_{i=1}^n A_i < L\right\}\right\rangle = p. \quad (\text{A.2})$$

To obtain Gates' explicit analytic solution, the often unrealistic simplifying assumption has to be made that large power units are more reliable than smaller ones. Then the solution can be graphed as a generalisation of the cost polygon method. Gates (1983) derived a second exact solution of the static optimal mix problem by assuming the load was exponentially distributed:

$$F_L(\ell) = 1 - e^{-\mu\ell} \quad (\text{A.3})$$

where  $\mu$  is constant. Although (A.3) is not realistic, the model has no restrictions on the relative reliabilities of large and small units, and offers some useful general insights.

Wind power can readily be incorporated into the model which utilizes (A.3) by replacing the random variable  $L$  by  $L - W$ , where  $W$  is the random variable representing wind power, which is assumed to have the following probability distribution, studied originally by Haslett and Carlin (1981):

$$F_W(w) \equiv \Pr(W \leq w) = \begin{cases} 1 - q \exp(-\nu w/w_r) & 0 \leq w < w_r \\ 1 & w = w_r \end{cases} \quad (\text{A.4})$$

where  $w_r$  is the rated wind power and  $\nu, q$  are dimensionless parameters.

Mathematical solutions for the reoptimised mix of a thermal grid containing wind power capacity  $W_r$  will be published separately.

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