

# Optimal thermal mix in electricity grids containing wind power

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*A numerical probabilistic model is used to calculate the optimal mix of thermal base and peak load plant in an electricity grid with zero storage and with differing amounts of wind power capacity. It is found that wind power capacity tends to displace conventional base load capacity with the same annual average energy production. The dollar value of the capital saving made by using wind power is often comparable with the dollar value of the fuel saving. The breakeven cost of wind power depends mainly on the capital cost of the base capacity, the fuel and other variable costs of the base plant, interest rates and on the penetration of wind power into the grid, but is relatively insensitive to other economic, grid, aerogenerator and load parameters.*

*Keywords: wind-electric power stations, thermal-electric power stations, electric power generation + reliability, mathematical models*

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## I. Introduction

The introduction of wind power capacity into a thermal electricity grid achieves two principle economies. First, wind energy saves fuel which would otherwise be burnt in conventional plant<sup>1-5</sup>. The amount and type of fuel saved is usually calculated by means of a dynamic computer simulation, based on hourly or half-hourly timesteps, in which the effective load at any given time equals the actual load minus the wind power<sup>3-5</sup>. Depending on the way the grid is operated, the fuel saved will be expensive (peak load) or cheap (base load), or a combination of both<sup>5</sup>.

Second, wind power capacity will substitute for some of the capacity of conventional power plant, given that the same total grid reliability is maintained<sup>3,6-11</sup>. To determine, by means of dynamic computer simulation, the sensitivity of the capacity credit of wind power to aerogenerator and grid parameters is difficult, because there are many variables and large amounts of computer time are required. However, when there is no storage in the grid, it is generally

a good approximation to discard the time dependence of wind power, load and outages of conventional units and, instead, to work with the probability distribution of each of these three random variables<sup>8,9</sup>. By using analytic approximations to these distributions, analytic expressions for the capacity credit can be derived in the limits of very small and very large wind penetrations, and valuable physical insights are thereby gained<sup>8</sup>. By using empirical, numerical distributions, numerical results which are consistent with a limited dynamic computer simulation are obtained<sup>9</sup>. In general, wind power capacity credit is found to be sensitive to the degree of penetration of wind power into the grid, to the unit sizes of conventional plant in the grid and, at large penetrations, to the cut-in speed of the aerogenerators<sup>9</sup>. For small penetrations, the wind power capacity credit, whether measured by the effective load carrying capability or by the equivalent firm capacity, is approximately equal to the average wind power<sup>8,9</sup>. These results for capacity credit are true even when there is no dispersal of aerogenerator sites into different wind regimes<sup>8,9</sup>.

To calculate approximately the total economic value of wind power in an electricity grid, it is necessary to determine whether the conventional capacity saved is expensive (base-load capacity) or cheap (peak-load capacity) and, at the same time, to determine the cost of fuel burnt in the base and peak plant in the new mix after the wind capacity is added.

The problem may be conceptualized as follows. A fixed amount of wind power capacity is added to the grid and various combinations of conventional base and peak load units, having the same capacity credit as the wind power capacity, are in turn retired from the grid. Given that the grid reliability, as measured by loss of load probability (LOLP), remains fixed, the problem is to determine what combination of base and peak plant should be retired to optimize the total annual economic savings (i.e. capacity plus fuel costs). Using an approximate analytic probabilistic model, Haslett<sup>12</sup> has found that wind power capacity substitutes mainly for base load plant.

In the present paper, a numerical probabilistic model is used to investigate, in quantitative detail, the nature of the

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Received: 17 July 1981, Revised: 19 January 1982

optimal mix of conventional plant after the introduction of wind power into an electricity grid. In Section II, the basic concepts are presented in mathematical form, and, in Section III, the numerical probabilistic model is described. The first principal result, reported in Section IV, confirms quantitatively Haslett's finding that wind power capacity substitutes for conventional base load capacity. To maintain the same LOLP, some additional peak load plant has to be installed, but this plant is rarely operated. The second principal result is that the dollar value of the capital savings made possible by wind power is often comparable to the dollar value of the fuel savings. In addition, the sensitivity of the breakeven cost of wind power to variation in the many parameters involved, including the relative annual economic costs of capacity and fuel and of different types of capacity, is determined.

## II. Basic concepts

The description of concepts and methods in this section is fairly general. In Section III, the more restricted assumptions and numerical values used in the calculations are described.

Consider an electricity grid containing a set of  $N$  thermal generating units with rated capacities  $c_i$ ,  $i = 1, \dots, N$ , with total rated capacity

$$C = \sum_{i=1}^N c_i$$

At a given time, the available capacity (i.e. that not undergoing planned or forced outage) of unit  $i$  is a random variable  $a_i$  (which is assumed here to take the values 0 or  $c_i$  only), and the total available capacity at a given time is

$$A = \sum_{i=1}^N a_i$$

The load or demand at a given time is the random variable  $L$ . A measure of the reliability of the generating system is the LOLP, denoted by  $p_0$ , which is the fraction of time the load  $L$  is greater than the total available power  $A$ :

$$p_0 = \Pr(A < L) \quad (1)$$

The value of  $p_0$  is determined by the electricity utility's choice of  $c_i$ ,  $N$  and hence  $C$ .

The static optimal mix of thermal units for a given value of  $p_0$  is the configuration of base, intermediate and peak load plant that minimizes the cost function

$$F = \sum_{i=1}^N c_i y_i + e_i z_i \quad (2)$$

Here  $y_i$  is the annualized capital cost per kilowatt of rated capacity  $c_i$ ,  $e_i$  is the annual usable energy generated by unit  $i$  and  $z_i$  is the total fuel cost (including operation and maintenance) per unit of energy generated. Note that  $F$  is only a measure of fixed and variable costs associated with power plant; it does not include transmission and distribution costs, overheads, etc. Also not considered here are optimal mixes for dynamic situations.

Assuming that the units are indexed in their merit order (i.e. unit  $i$  is used before unit  $i + 1$ , because the former's operating cost is lower), the annual energy  $e_i$  generated by unit  $i$  is given by

$$e_i = \bar{L}_{i-1} - \bar{L}_i + \bar{\epsilon}_i \quad i = 1, \dots, N \quad (3)$$

Here,  $L_0 \equiv L$ ,  $\bar{L}_i$  is the expectation value of  $L_i$ ,  $\epsilon_i$  is the excess energy generated (explained below) and the  $L_i$  are calculated iteratively using

$$L_i = \begin{cases} L_{i-1} - a_i & L_{i-1} > a_i \\ 0 & L_{i-1} < a_i \end{cases} \quad i = 1, \dots, N \quad (4)$$

where  $L_i$  and  $a_i$  are random variables. In other words,  $L_1$  is the effective load after the availability  $a_1$  of the first unit (highest in the merit order, lowest fuel cost) is subtracted from the load  $L$ ;  $L_2$  is the effective load after the availability  $a_2$  of the unit second in the merit order is subtracted from the effective load  $L_1$ ; and so on. Energy is consumed by the unit  $i$  when there is sufficient load  $L_{i-1}$  remaining to take up the available capacity.

In an electricity grid, it is possible that, at times, more power may be generated than can be absorbed by users, even when all units are regulated to their minimum power output. This is especially likely when wind capacity is incorporated into the grid in the absence of storage, since the rated wind power  $W_r$  is typically 2-4 times the average wind power output  $\bar{W}$ . Furthermore, the wind power  $W$  is produced irrespective of the load, whereas most conventional plant can be run below rated power when necessary. A variation of the procedure indicated by equation (4) can be used to calculate the amount of excess energy  $\epsilon_i$  generated and associated with each unit  $i$ . The cost of this lost or wasted energy is included in the cost function  $F$ .

In the calculation of  $\epsilon_i$  it is assumed that all base load units are always fired up, with unit  $i$  producing a minimum power  $b_i$ . This is clearly unrealistic when the mix includes a preponderance of base load units but results in only small losses  $\epsilon_i$  when  $W = 0$ , since the calculation allows all the units to be regulated to their minimum power outputs simultaneously. When  $W > 0$ , the  $\epsilon_i$  increase (because there is also then an energy loss  $\epsilon_W$  associated with  $W$ ), and this is the major cause of the reduced economic value of marginal additions of wind power capacity when  $\bar{W}/\bar{L}$  is sizeable. Note also that the calculation does not take into account the fuel burnt in keeping plant on standby.

For conventional plant, planned outages are assumed not to contribute to the LOLP in our model. This means that when a loss of load event occurs, no units are undergoing planned outage.

For wind power, both forced and planned outages due to mechanical failures are incorporated by a scaling factor applied to the capital cost of wind power plant. Since any single wind power unit is only a small fraction of the entire wind capacity, the effect of maintenance and mechanical failures on a wind farm is to slightly reduce its average wind capacity. The wind power plant outage rate is assumed to be 5%. The mean wind power output  $\bar{W}$  is a fraction  $f_W$  of the rated wind capacity  $W_r$ ; the capacity factor  $f_W$  depends primarily on the choice of the ratio

$R = v_r/\bar{v}$ , where  $v_r$  is the rated wind speed of the aerogenerators and  $\bar{v}$  is the annual average wind speed<sup>13</sup>.

### III. Numerical probabilistic model

A real electricity grid has a wide variety of plant and fuel characteristics. Our model simplifies this complex system by assuming that there are only two categories of conventional power plant, 'base' and 'peak', characterized by their respective capital costs, fuel costs and ranges of power regulation.

In effect, equation (2) becomes

$$F(N_b) = N_b c_b y_b + N_p c_p y_p + z_b \sum_{i=1}^{N_b} e_i(N_b) + z_p \sum_{i=N_b+1}^{N_p} e_i \quad (5)$$

where the subscripts  $b$  and  $p$  denote 'base' and 'peak' units and  $N_b$  and  $N_p$  are the numbers of identical base and identical peak units, where  $N_b + N_p = N$ ,  $N_b c_b + N_p c_p = C$  and  $C$  is fixed by equation (1). The solution to the optimal mix problem is then the value of  $N_b$  that minimizes  $F$ . Equations (3) and (4) mean that some of the energies  $e_i$  generated by base load units will be a function of  $N_b$ , and this is indicated in equation (5).

In contrast, the analytic probabilistic model of Haslett<sup>12</sup> considers, instead of equation (5), an equation for a related cost function,  $F^*(C_b)$ , where  $C_b = N_b c_b$  takes continuous values, in which the total base load energy generated has the form

$$\int_0^{C_b} H(x) dx$$

Haslett's cost function  $F^*(C_b)$  is a linear function of  $C_b$  and is readily optimized, yielding analytic approximations which provide qualitatively valuable results<sup>12</sup>.

The present numerical probabilistic model deals with the nonlinear function of a discrete variable  $F(N_b)$  and yields quantitatively useful results in Section IV.

Wind power is specified by its rated or installed capacity  $W_r$ , of which power  $W$  (a random variable) is available at a given time. The redetermination of the optimal mix in the presence of wind power plant is a straightforward generalization of equation (3) to equation (5) in which  $L - W$  replaces  $L$ .  $\Delta N_b$  thermal base load units are retired and  $\Delta N_p$  new peak load units (where  $\Delta N_p < \Delta N_b$ ) are installed, with the result that the total installed thermal capacity becomes

$$C^* = \sum_{i=1}^{N - \Delta N_b + \Delta N_p} c_i \quad (6)$$

and the equation for LOLP is now

$$p_0 = \Pr(A^* + W < L) \quad (7)$$

where  $A^*$  is the available conventional capacity corresponding to the reduced conventional capacity  $C^*$ . Then  $C - C^*$  is a measure of the capacity credit of wind power and is known as the capacity saving<sup>8</sup>.

The numerical probabilistic model is based on a mesh of power intervals (typically 4 MW intervals in a grid with a total rated capacity of 2 400 MW) on which the random variables  $A$ ,  $L$ ,  $W$ , etc. are represented. The numerical probability distribution (or density) function of each random variable is given by the histogram of values of that variable on the mesh.  $C^*$  is obtained by solving numerically equation (7), in which  $p_0$  is a given constant and the 'sums' or 'differences' of the random variables (e.g.  $L - W$ ) become numerical convolutions of their respective probability distribution functions. In practice, the value of  $C^*$  required to satisfy equation (7) is obtained from a sequence of trial values of  $C^*$ . The optimal mix for a given set of capital and fuel costs  $y_b, y_p, z_b, z_p$  is found by determining the numbers of base and peak units with capacity resulting in the appropriate LOLP and using energies  $e_i$  which give the minimum cost in equation (5).

Because the capacity saving  $C - C^*$  due to wind power is not, in general, an integral multiple of the unit capacity  $c_b = c_p$ , in practice there is usually one peak unit at the bottom of the merit order with a rated capacity less than the other conventional units, so as to satisfy equation (7).

There are a large number of parameters in the model, all of which can affect the results. To further delineate the model, the 'standard' values of these parameters will now be specified. Alternative values or assumptions used to test the sensitivity of the results to the parameters will be mentioned in Section VI.

The standard grid is taken to have  $N = 24$  units, each with a rated capacity of  $c_i = 100$  MW. The forced outage rate of each unit is  $f_i = 0.10$  and the planned outage rate is  $g_i = 0.10$ . (Here 'rate' means the fraction of time spent in the indicated outage condition.) Hence, for a single unit at the top of the merit order, the mean available power  $\bar{a}_i = 80$  MW or 8% of the average load  $\bar{L} = 1000$  MW. Each unit can be either a base or a peak unit. Apart from different capital and fuel costs, the base and peak units differ only in that the base units can be regulated down to a minimum of  $b_i = 40$  MW unless undergoing outage, while the peak units can be regulated to zero output.

The probability distribution of load  $L$  is taken from half-hourly load data from the Western Australian grid for the period 1970-79, rescaled to the 1978 mean level by fitting a linear increase in annual mean load over the 10 year period, then rescaled again so that  $\bar{L} = 1000$  MW. The standard deviation is then 329 MW. The LOLP is  $p_0 = 2.0 \times 10^{-4}$ .

The wind speeds were chosen to follow a Rayleigh distribution with mean (at hub height) of  $\bar{v}$ . The probability distribution of available wind power  $W$  is obtained by filtering the wind speeds through the aerogenerator response function. The 'standard' aerogenerators have a cut-in speed  $v_s = 0.53 \bar{v}$ , a simple cubic response from  $v_s$  up to the rated speed  $v_r = 1.5 \bar{v}$  and a constant response from  $v_r$  up to the furling speed  $v_f = 3.3 \bar{v}$ . These parameter values, taken together with the wind plant outage rate, give a

capacity factor of 0.36. It is assumed that wind speeds are homogeneous throughout the region of wind power production. This assumption underestimates the capacity credit of wind power.

There are five basic costs in the problem: the capital costs of base, peak and wind capacity, and the fuel costs of operating base and peak plant. Since all the prices can be scaled if desired, the annual cost of base load capacity was set at the arbitrary value of  $\$100 \text{ kW}^{-1} \text{ y}^{-1}$ , where kW denotes 'rated kW'. This value represents, for example, the real annual charge (real interest plus depreciation) on a capital investment of  $\$1000$  per rated kW at 10% per annum. The cost of peak load capacity was set at 1/2, 1/4 or 1/8 of the base load capacity cost. The base fuel cost was set at  $0.57 \times 2^m \text{ c/kWh} = \$50 \text{ 2}^m (\text{kW}\cdot\text{year})^{-1}$ ,  $m = 0, 1, 2, 3$ , and the peak fuel cost at 2, 4 and 8 times this value.

These 36 cost ranges span a variety of likely capital and fuel cost ratios. The capital cost of wind power capacity was obtained from the model as the annual cost for wind power capacity which gives the fuel and capital savings by wind that are just equal to its capital cost.

Note that the same numerical value of the wind power capacity credit (in MW) is obtained irrespective of the mix of base and peak plant assumed. In principle, the wind power capacity credit can be in the form of base or peak plant or any linear combination thereof, and likewise the fuel saved can be shifted to base or peak fuel by specifying the mix. The optimal mix is determined only by the costs specified for the base and peak capacity and fuel.

This conclusion was verified by means of a half-hour by half-hour simulation model, which has been used elsewhere

**Table 1. Optimal mixes of thermal base and peak plant, and total costs, for 36 sets of capital and fuel costs and zero wind power capacity in 'standard' grid model**

Cost set identity no.	Annual cost of capacity, $\$ \text{ kW}^{-1} \text{ y}^{-1}$		Fuel cost, $\$ (\text{ kW y})^{-1}$		Optimal mix No. of units		Costs in $\text{M}\$ \text{ y}^{-1}$		
	base	peak	base	peak	base	peak	capital	fuel	total
1	100	50	50	100	0.0	24.0	120	100	220
2	100	50	50	200	13.7	10.3	188	66	254
3	100	50	50	400	16.8	7.2	204	61	266
4	100	50	100	200	10.5	13.5	173	124	297
5	100	50	100	400	16.3	7.7	202	112	314
6	100	50	100	800	18.4	5.6	212	111	323
7	100	50	200	400	14.9	9.1	194	214	409
8	100	50	200	800	17.9	6.1	210	213	423
9	100	50	200	1 600	19.7	4.3	218	214	432
10	100	50	400	800	16.7	7.3	204	415	619
11	100	50	400	1 600	19.1	5.0	215	418	633
12	100	50	400	3 200	20.6	3.4	223	421	644
13	100	25	50	100	0.0	24.0	60	100	160
14	100	25	50	200	10.5	13.5	139	86	225
15	100	25	50	400	15.6	8.4	177	69	246
16	100	25	100	200	6.0	18.0	105	152	257
17	100	25	100	400	15.0	9.0	172	121	293
18	100	25	100	800	17.5	6.5	191	117	308
19	100	25	200	400	13.0	11.1	157	227	384
20	100	25	200	800	17.1	6.9	188	218	406
21	100	25	200	1 600	19.0	5.0	202	218	420
22	100	25	400	800	15.8	8.2	178	421	599
23	100	25	400	1 600	18.4	5.6	198	422	620
24	100	25	400	3 200	20.1	3.9	211	424	635
25	100	12	50	100	0.0	24.0	29	100	129
26	100	12	50	200	8.9	15.1	107	99	206
27	100	12	50	400	15.0	9.0	161	74	235
28	100	12	100	200	0.0	24.0	29	200	229
29	100	12	100	400	14.3	9.7	155	126	281
30	100	12	100	800	17.2	6.8	180	120	300
31	100	12	200	400	11.7	12.3	132	236	369
32	100	12	200	800	16.7	7.4	175	222	397
33	100	12	200	1 600	18.7	5.3	193	220	414
34	100	12	400	800	15.3	8.7	163	425	588
35	100	12	400	1 600	18.2	5.8	189	424	613
36	100	12	400	3 200	19.9	4.1	204	426	630

'kW' denotes rated kilowatt, 'kW<sub>y</sub>' denotes 'kilowatt-year' = 8 760 kWh = 31.54 GJ

to obtain information about fuel savings and grid operating strategies<sup>5</sup>. By setting up the simulation model appropriately, the results of the numerical probabilistic model can be reproduced almost exactly, for both fuel savings and capacity credit.

#### IV. Results for 'standard model'

For each set of prices, the optimal mix of base and peak plant was obtained by fitting a parabola to the three lowest cost configurations and using the vertex of the parabola to fix the optimal mix and the minimum cost. Thus noninteger values of  $N_b$  and  $N_p$  will in general be obtained at the optimum. The results are listed in Table 1. Each row in Table 1 represents a different cost set  $\{y_b, y_p, z_b, z_p\}$  whose values are given in columns 2-5 and whose identification number is given in column 1.

When wind power capacity is introduced into the grid, a new optimum mix is obtained which invariably has a smaller number of base units. The number of peak units is adjusted up or down so that the LOLP remains at  $p_0$ .

Table 2 lists, for each of the 36 cost sets specified in Table 1, the optimal mix and capacity and fuel savings for a grid in which sufficient wind power capacity is introduced to satisfy 8% of the average load ( $\bar{W}/\bar{L} = 0.08$ ). That is, the average wind power output has been chosen to be equal to the average power output of a single conventional base load unit at the top of the merit order. Also listed are the breakeven costs of wind power, i.e. the annualized capital costs of wind power capacity such that the capital cost of wind power equals the dollar value of capital and fuel savings due to wind power. According to this definition, the larger the breakeven cost, the more favourable the economics of wind power.

**Table 2. Savings in capacity and fuel in reoptimized 'standard' grid containing wind power capacity ( $\bar{W}/\bar{L} = 0.08$ ), relative to optimal mix with  $\bar{W} = 0$  (Table 1), for 36 sets of capital and fuel costs**

Cost set identity no.	Savings of units		Savings in M\$ $y^{-1}$			Breakeven wind power cost, \$ $kW^{-1}y^{-1}$
	base	peak	capital	fuel	total	
1	0.00	0.53	2.7	8.0	10.7	48
2	1.10	-0.57	8.2	3.0	11.2	51
3	0.98	-0.44	7.6	3.0	10.6	48
4	1.07	-0.54	8.0	7.4	15.4	70
5	1.04	-0.51	7.9	6.4	14.3	65
6	0.96	-0.43	7.5	6.2	13.7	62
7	1.23	-0.70	8.8	13.3	22.2	101
8	1.03	-0.49	7.8	12.9	20.8	94
9	0.95	-0.42	7.4	12.6	20.0	91
10	1.22	-0.68	8.7	26.9	35.6	162
11	1.04	-0.51	7.9	25.8	33.7	153
12	0.97	-0.44	7.5	24.9	32.5	147
13	0.00	0.53	1.3	8.0	9.3	42
14	1.02	-0.48	9.0	3.6	12.5	57
15	1.01	-0.48	8.9	2.8	11.7	53
16	1.13	-0.60	9.8	6.8	16.6	76
17	1.08	-0.54	9.4	6.2	15.6	71
18	0.96	-0.43	8.5	6.3	14.8	67
19	1.23	-0.70	10.6	13.4	23.9	109
20	1.03	-0.50	9.1	12.9	22.0	100
21	0.95	-0.42	8.5	12.6	21.0	96
22	1.22	-0.69	10.5	26.8	37.3	169
23	1.03	-0.49	9.0	25.9	34.9	159
24	0.97	-0.43	8.6	25.0	33.6	152
25	0.00	0.53	0.6	8.0	8.6	39
26	0.99	-0.46	9.4	3.8	13.1	60
27	1.03	-0.49	9.7	2.7	12.4	56
28	0.00	0.53	0.6	16.0	16.6	75
29	1.10	-0.56	10.3	6.0	16.3	74
30	0.97	-0.44	9.2	6.1	15.3	70
31	1.16	-0.63	10.8	14.0	24.8	113
32	1.04	-0.51	9.8	12.8	22.6	103
33	0.95	-0.42	9.0	12.6	21.6	98
34	1.22	-0.69	11.4	26.8	38.2	173
35	1.03	-0.49	9.7	25.9	35.6	161
36	0.96	-0.43	9.1	25.1	34.1	155

It is evident that the relative fraction of economic savings due to capacity savings by wind power reflects the relative costs of capacity and fuel. For some cost sets, the capacity savings are greater than the fuel savings. Clearly, the breakeven cost of wind power depends sensitively on

- the fuel cost of thermal base load plant (e.g. comparison of cost sets 3, 5 and 7)
- the capital cost of thermal base load capacity (e.g. comparison of the breakeven cost of wind power in cost set 8 with double the breakeven cost in cost set 17 and with four times the breakeven cost in cost set 26).

However, the breakeven cost of wind power is relatively insensitive to the capital and fuel costs of thermal peak load plant.

## V. Example

The results can be readily scaled in several ways. If the cost of base load capacity is changed from  $\$100 \text{ kW}^{-1} \text{ y}^{-1}$  to  $\$x \text{ kW}^{-1} \text{ y}^{-1}$  and all other costs are changed by the same ratio  $x/100$ , the results are unchanged. If the grid size is changed from that of the 'standard' grid with  $\bar{L} = 1000 \text{ MW}$  to  $\bar{L} = X \text{ MW}$  and all other quantities measured in MW are changed by the ratio  $X/1000$ , then the results are unchanged.

Scaling up to grids with much larger units will require a small correction resulting from changes in the forced outage rate. For instance, for a grid with an installed capacity of 14 400 MW and  $N = 24$ , the installed capacity per unit is 600 MW and a forced outage rate of about 0.16 would be more appropriate than 0.1; for units of 1000 MW, the forced outage rate could reach 0.2.

In the following example, it is assumed that  $N = 24$ , that constant 1980 dollars are used, that the discount rate (in real terms) is 5% p.a. and that the lifetime of all types of power plant is 25 years. Then the real annual charge, expressed as a levelled annuity, is 7.1%. Wind power plant attracts an additional real annual charge of 2% to cover operation and maintenance, giving a total real annual charge of 9.1%. (The O & M costs of thermal power plant are included in their total 'fuel' costs.) Capital costs include interest during construction. 'kW' denotes 'rated kilowatt'.

A grid with base load units having a capital cost of  $\$100 \text{ kW}^{-1} \text{ y}^{-1}$  or  $\$1408 \text{ kW}^{-1}$  and a fuel cost of 1.3 c/kWh or  $\$100/\text{kWy}$  and peak load units having a capital cost  $\$25 \text{ kW}^{-1} \text{ y}^{-1}$  and a fuel cost of  $5.2 \text{ c(kWh)}^{-1}$  or  $\$400 (\text{kWy})^{-1}$ , has an average demand  $\bar{L} = 1000 \text{ MW}$ . From Table 1, the model grid corresponds to cost set number 17. We assume that a forced outage rate of 0.10 is adequate to a first approximation and so Table 2 can be used.

In the presence of a wind penetration,  $\bar{W}/\bar{L}$ , of 8%, the optimal number of base units is reduced from 15.0 to 13.9 while the number of peak units is increased from 9.0 to 9.54. The annual capital saving is  $\$9.4 \text{ M}$ , which is 50% greater than the fuel saving of  $\$6.2 \text{ M}$ . The breakeven cost for wind power,  $\$71 \text{ kW}^{-1} \text{ y}^{-1}$ , corresponds to  $\$780 \text{ kW}^{-1}$ , a figure that is slightly below the estimated future cost (in 1980 dollars) of about  $\$800$  to  $\$1000 \text{ kW}^{-1}$  for MOD-2 in mass production. In other words, wind power is not quite economically competitive at this cost set.

However, if fuel prices double in real terms, while the capital costs and the ratio of coal to oil fuel prices remain the same, the model grid corresponds to cost set 20, for which the breakeven cost of wind power is  $\$100 \text{ kW}^{-1} \text{ y}^{-1}$  or  $\$1099 \text{ kW}^{-1}$ . This is likely to be economically competitive in mass production.

If the base capacity cost in cost set 17 is doubled and fuel and other variable costs remain the same, then cost set 26 may be used, provided that the breakeven cost is doubled. Thus the breakeven cost of wind power rises to  $\$120 \text{ kW}^{-1} \text{ y}^{-1}$ , or  $\$1319 \text{ kW}^{-1}$ , which may already be economically competitive for at least one type of commercial medium-sized aerogenerator.

## VI. Sensitivity of results to variations in grid and wind power parameters

The results given in Section IV were based on a model with a 'standard' set of grid and wind power parameters, the values of which were specified in Section III. The effect on the breakeven costs of wind power of varying each of these parameters separately will now be summarized. Because the numerical probabilistic model has such a short computation time, it has been possible to perform the sensitivity analysis for each of the 36 cost sets. However, in the interests of brevity, only the general trends are summarized here.

As the penetration of wind power  $\bar{W}/\bar{L}$  into the grid is increased from 8% to 40%, the breakeven cost of wind power decreases typically by about 25%, as a consequence of reductions in both the marginal fuel saving and marginal capacity credit of wind power. However, in the real world, this decrease would be offset to some extent by the reduced cost of aerogenerators in large scale mass production.

The breakeven costs of wind power are also very sensitive to the value chosen for  $v_r/\bar{v}$ , given that  $\bar{W}$  is kept constant. For instance, increasing  $v_r/\bar{v}$  from 1.5 to 2.0 has the effect of nearly halving the breakeven costs. This result follows inevitably from the assumption that the capital cost of an aerogenerator is determined by its rated power.

The following parameter changes have a relatively small effect (typically less than about 10%) on the breakeven cost of wind power:

- decreasing the LOLP  $p_0$  by a factor of 100 (by adding peak load units to the grid),
- increasing or decreasing, by a factor of 2, the forced and planned outage rates of thermal units,
- reducing the permitted range of regulation of the output of thermal units to zero,
- changing the cut-in wind speed of aerogenerators from  $0.53\bar{v}$  to  $0.8\bar{v}$ ,
- replacing the Rayleigh wind speed distribution by an actual wind speed distribution measured at Fremantle, Western Australia,
- doubling or halving the standard deviation of the load,

- replacing the 24 × 100 MW units comprising the 'stand-ard' grid by a smaller number of larger units.

It should be noted that, although the above parameter changes do not alter significantly the breakeven cost of wind power in a thermal grid which is optimally mixed, some of them do have significant effects on the fuel saving<sup>5</sup> or capacity credit<sup>9</sup> (measured in megawatts). As a consequence, they may have a considerable effect on the economics of wind power in a grid that is not optimally mixed.

## VII. Conclusion

When wind power capacity is introduced into an electricity grid and a new optimal mix of base and peak conventional plant is obtained with the same overall grid reliability, less fuel is burnt in the conventional plant and less conventional capacity is required. In other words, wind power capacity saves fuel and has a capacity credit. The economic value of these fuel and capacity savings depends on base and peak capacity and fuel costs.

A numerical probabilistic model has been used to calculate optimal mixes of thermal base and peak plant with and without wind power capacity. It is found that wind power capacity displaces that amount of base plant that has approximately the same average power output as the wind power plant. To maintain the same grid reliability, as measured by LOLP, a small amount of thermal peak load plant has to be installed as well, but this peak plant rarely has to be operated and so plays the role of a reliability insurance with a low premium.

The most important factors determining the breakeven cost of wind power are the capital cost of thermal base capacity, the cost of base load fuel, the penetration of wind power capacity into the grid, the ratio  $v_p/\bar{v}$  and interest rates. The breakeven cost is relatively insensitive to other economic, grid and aerogenerator parameters, provided that the grid has an optimal mix.

For some cost sets, the dollar value of the savings in thermal plant capacity produced by wind power capacity is comparable in magnitude with the dollar value of the savings in fuel.

The static optimal mix has been used as a means of comparing the economic value of wind power with that of conventional thermal plant. The method has the advantage over a simple marginal cost comparison of alternative power units that it takes account of the effect of planning choices on the reliability and energy production of the whole grid. Furthermore, the optimal mix method offers directions for generation planning, even when the existing grid mix is not optimal.

## VIII. Acknowledgements

This work is supported by a grant from the Australian National Energy Research, Development and Demonstra-

tion Council (NERDDC). The authors wish to thank John Carlin for helpful discussions.

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