An Integrative Model of Rational Diet and Physical Activity:
Physiological, Gastronomic and Budgetary Aspects

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This paper constructs a model for analyzing the deviations of consumers’ diet and physical activity from their physiologically optimal ones with distinction between a nutritionally and digestively superior food and a taste and price superior food. The consideration of both diet and physical activity and the inclusion of cause-and-effect relationships of the deviations from their physiological optimal ones with ageing, craving, digestive discomfort, health-dependent budget, and non-food consumption and the consideration of intertemporally bounded rationality adds realistic features to the analysis of rational eating.

Keywords: Health; Diet; Exercising, Ageing; Craving; Budget; Bounded Rationality

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1. Introduction
Earlier attempts to model the difference between rational eating and physiologically optimal eating have focused on quantity (Levy, 2002a; Dragone, 2009; Levy 2009). In these attempts, the steady-state combination of food-consumption and weight of an expected lifetime-utility-maximizing consumer has been found to be physiologically excessive, reinforced by customs and habits, asymptotically unstable, yet approachable from two opposite directions. These trajectories of rational convergence to a physiologically excessive steady state have highlighted the difficulty in overcoming overweight, obesity and their associated diseases. Eating and health are also affected by qualitative characteristics of food. Some types of food have opposite effects on the consumers’ instantaneous utility and health: their consumption generates high instantaneous utility, but deteriorates health and, subsequently, productivity, income and future utility (Levy, 2002b, 2006). Levine et al.’s (2003) study on the neurobiology of preference has shown that central regulatory mechanisms favor foods containing sugar and fat over other nutrients. Having a high concentration of these substances makes physiologically harmful types of food taste-appealing and, possibly, addictive. Due to cheaper ingredients, easier preparation process and storage and value of time, some such food-products are often less expensive than their healthier substitutes (Philipson and Posner, 1999; Lakdawalla and Philipson, 2002; Drenowski, 2003). The size of the fast-food and snack-food industries suggests that the actual diet of many consumers deviates significantly from the physiologically optimal strategy of abstinence. Taxing less healthy, cheaper food-products may improve public health. Due to the regressive nature of the tax, the alternative policy measure of subsidizing the healthier substitutes should be assessed (Lordan and Quiggin, 2011). Furthermore, the choice of food and the effectiveness of policy instruments also depend on the consumer’s physical activity. Yaniv et al. (2009) have argued that implementation of a fat-tax reduces obesity among non-weight-conscious consumers, but not necessarily among weight-conscious consumers.

The objective of this paper is to construct a detailed, integrative model of the divergence of the consumer’s diet and physical activity from the physiologically optimal ones. The model facilitates the exploration of how the deviations from the physiologically optimal diet and physical activity are affected by availability and affordability of a taste-superior food alongside a nutritionally and digestively superior food, by diet and physical activity dependent ageing, by health and physical activity dependent income, by utility from food, other goods and physical activity, and by changes in craving for the taste-superior food.
The model takes into account that craving can be moderated, and even inverted, by recurring episodes of indigestion.

The proposed model takes into account intertemporarily bounded rationality. The alternative paradigm, lifetime-utility maximization, has been considered in the aforementioned, less complex, models of rational eating to demonstrate an overweightness trap even for a self-controlled and analytically sophisticated consumer. As in Yaniv et al. (2009), the complexity of the proposed model makes intertemporarily bounded rationality a sensible choice. The assumed intertemporarily bounded rational agent maximizes current utility with some consideration of the implications of the current diet and physical activity for her condition—craving for food rich in flavoring ingredients and health.

The novelty of the proposed model is in treating the quantity and quality of food and the time spent in physical activity as endogenous variables and in the variety and formulation of their determinants. The determinants include physiological aspects, gastronomic aspects and budgetary aspects of diet and physical activity. They are formulated in sections 2, 3 and 4. Section 5 presents an intertemporarily bounded rational objective function, computes the diet and length of physical activity that maximize this function and discusses their properties. The introduction of different types of food, physical activity, craving, ageing, productivity, income and bounded rationality adds realistic features to the analysis of food-consumption. The introduction of ageing eliminates steady states—the focus of the earlier studies on rational eating.

2. Physiological aspects of diet and physical activity

In the proposed model food is divided into two types: nutritionally and digestively superior food and taste-superior food. A finer classification is possible, but analytically less manageable. These two types are nicknamed “healthy food” and “junk food”, respectively. The distinction between junk food and healthy food depends on the concentration of calories, fat and flavoring ingredients, whose presence in the human body beyond a critical level is harmful. In addition to a high concentration of these substances, junk food is lacking vital nutrients such as fibres and vitamins. Healthy food is denoted by $h$ and junk food by $j$. Their quantities in the consumer’s diet, $c_h$ and $c_j$, are measured in units of weight, say grams.

Let $0 < x(t) \leq 1$ denote the consumer’s physical activity rate at $t$—the fraction of (the day) time spent in physical activity (rest and sleep time is assumed, for simplicity, to be constant). Healthy food is physiologically essential. As long as healthy food is available, junk
food is not physiologically essential. Let \( c_{h}^{sed} \) and \( c_{h}^{act} \) be, respectively, the number of grams of healthy food required for maintaining a completely sedentary consumer (\( x=0 \)) and a fully physically active consumer (\( x=1 \)) in the best possible health. Assuming linear relationship between physiologically optimal nutrition and physical activity rate, the physiologically optimal consumption of healthy food by a consumer with physical activity rate \( x \) at \( t \) is \( c_{h}^{sed} + (c_{h}^{act} - c_{h}^{sed})x(t) \). Consumption of a larger quantity leads to a loss of health. Assuming that healthy food is available, the number of grams of junk food required for maintaining the consumer in the best possible health is zero. The combination \( ((c_{h}^{sed} + (c_{h}^{act} - c_{h}^{sed})x(t)),0) \) is the physiologically optimal diet at age \( t \) for a consumer with physical activity rate \( x(t) \). It has the highest nutritional value for that consumer. Denoting the actual number of grams of healthy food and the actual number of grams of junk food consumed at \( t \) by \( c_{h}(t) \geq 0 \) and \( c_{j}(t) \geq 0 \), respectively, the consumer’s actual diet at age \( t \) is \( (c_{h}(t),c_{j}(t)) \).

The consumer’s health at any age \( t \) is represented by \( H(t) \geq 0 \). Her state of health at birth is \( H_0 > 0 \). Due to the inevitable physiological decay there is an upper bound, \( T_{max} \), on the consumer’s life-expectancy, \( T \), which can be reached by maintaining the best possible health. As long as the consumer adheres to the physiologically optimal diet and full physical activity, \( H(t) \) is equal to her best possible health. Ageing is represented by \( t/T_{max} \in [0,1] \) and is accelerated by deviations from the physiologically optimal diet and from full physical activity. The negative effect of the accelerated ageing on health is
\[
-\delta[1+\delta_x(1-x(t)) + \delta_h(c_{h}(t)-(c_{h}^{sed} + (c_{h}^{act} - c_{h}^{sed})x(t)))^2 + \delta_j(c_{j}(t)-0)^2](t/T_{max}).
\]
The scalar \( \delta > 0 \) is the marginal health-degradation engendered by ageing under full physical activity and physiologically optimal diet. The scalar \( 0 < \delta_x < 1 \) indicates the direct marginal contribution of sedentary \( (1-x(t)) \) to ageing. The positive scalars \( 0 < \delta_h < 1 \) and \( 0 < \delta_j < 1 \) denote the consumer’s physiological sensitivity to deviations of the actual intake from the physiologically optimal quantities of healthy food and junk food, respectively. For simplicity, symmetry is assumed and the possible interaction effect of the two types of deviations is ignored.

Beyond a critical age ageing dominates regeneration and health diminishes. Death occurs when \( H \) reaches 0. The consumer’s best possible health changes at a rate that declines from a positive regeneration rate, \( r_b \), at birth \( (t = 0) \) to -1 at the moment of death.
\( t = T_{\text{max}} \). Assuming inactivity at that furthest moment of death \( (x(T_{\text{max}}) = 0) \), the health-motion equation can display the said rates of change by setting \( \delta \) to be equal to \( (1 + r_b) \):

\[
\dot{H}(t)/H(t) = r_b
\]

\[-(1 + r_b)[1 + \delta x(1 - x(t)) + \delta_h (c_h(t) - (c_{h}^{\text{sed}} + (c_{h}^{\text{act}} - c_{h}^{\text{sed}})x(t)))^2 + \delta_j (c_j(t) - 0)^2] (t/T) \]

Equation (1) also displays the following properties. Death (that is, \( \dot{H}/H \leq -1 \)) is inevitable. For a consumer adhering to full physical activity and the corresponding physiologically optimal diet, \( (c_h^\text{o},0) \), ageing (i.e., \( t/T_{\text{max}} \)) adversely affects the rate of change of health at a rate \( (1 + r_b) \) that exceeds the initial regeneration rate and hence health peaks at age \( [r_b / (1 + r_b)]_T_{\text{max}} \). Thereafter, ageing dominates regeneration and, consequently, health deteriorates and is completely eroded at \( T_{\text{max}} \) (i.e., \( \dot{H}(T_{\text{max}})/H(T_{\text{max}}) = -1 \)). For a consumer diverging from full physical activity and the corresponding physiologically optimal diet, \( (c_h^\text{o},0) \), ageing is faster and death occurs earlier.

3. Gastronomic aspects of diet and physical activity

While health is affected by the nutritional value of the diet, enjoyment of food is determined by taste and digestive comfort. Due to a high concentration of flavouring substances, junk food is tastier than its healthier substitute. This property is expressed by letting the taste of healthy food be equal to 1 and the (relative) taste of junk food be indicated by \( \alpha > 1 \). However, beyond a critical level of junk-food consumption the overdose of the flavoured, fat-rich and fibre-poor food causes discomfort (e.g., nausea, gastro-oesophageal reflux and constipation). Due to a larger calorie requirement and faster metabolism the critical level rises with physical activity rate and given by \( x(t)\bar{c}_j \geq 0 \), where the scalar \( \bar{c}_j > 0 \) indicates the digestive discomfort threshold of a fully physically active consumer. The larger the overdose \( (c_j - x(t)\bar{c}_j) \), the stronger the digestive discomfort. The discomfort experienced at \( t \) intensifies the consumer’s aversion to junk food and thereby moderates its future consumption. Hence, it is possible that the present junk-food consumption of a consumer with a strong relative taste for junk food, but a sensitive digestive system, is moderated significantly by past overdosing. In contrast, when the consumption of junk food is smaller than \( \bar{c}_j \), the discomfort-free taste intensifies craving for junk food due to its addictive flavouring ingredients. It is therefore possible that the present junk-food consumption of a consumer with a weak relative taste for junk food is increased by moderate past consumption.
Due to digestive comfort, or discomfort, the consumer’s current attraction (-1 ≤ A(t) ≤ 1) to junk food evolves from an initial state of unfamiliarity-based indifference (A(0) = 0) to craving (0 < A ≤ 1), or aversion (-1 ≤ A < 0), interchangeably. Aversion diminishes the consumer’s pleasure from eating junk food, whereas craving intensifies. With this argument in mind, the absolute value of the change in the consumer’s state of aversion (craving) to junk food is assumed to rise with overdosing (under-dosing), but in a rate that diminishes with the already existing intensity:

\[ \dot{A}(t) = -\theta [1 - A(t)]^2 \left\{ \frac{[c_j(t) - x(t)c_j]}{c_j} \right\}. \] (2)

The scalar \(0 < \theta \leq 1\) reflects the sensitivity of the consumer’s digestive system to junk food. Starting life with unfamiliarity-based indifference to junk food (A(0) = 0), equation (2) ensures that -1 ≤ A(t) ≤ 1 for every \(t \in [0, T^*]\).

To display pleasure from eating, the quantities of junk food and healthy food consumed at \(t\) will be introduced into the consumer’s utility function as a weighted sum (m):

\[ m(t) = \frac{\alpha}{1 - A(t)} c_j(t) + c_h(t). \] (3)

This weighted sum reflects that neither junk food, nor healthy food, is gastronomically essential. The ratio \(\alpha / [1 - A(t)]\) indicates the consumer’s relative marginal pleasure from the junk-food component of her diet at age \(t\). If, for example, by age \(t\) the consumer has developed some aversion to junk-food (-1 ≤ A(t) < 0) through past episodes of overdosing discomfort, her relative marginal pleasure from junk food at \(t\) is smaller than the relative taste of junk food (\(\alpha\)).

4. Budgetary aspects of diet and physical activity and instantaneous utility

Health affects the consumer’s productivity and, in turn, budget. Recalling equation (1), the consumer’s budget is indirectly affected by her past and present diets through their effects on her health. Knowledge and experience determine the current rate of return, \(w(t)\), on the consumer’s health. Since knowledge and experience are accumulated over time, \(w\) is taken to be growing over the lifespan, for simplicity, at a constant rate \(\gamma\):

\[ w(t) = \gamma t. \] (5)

Typical to technologically advanced economies, the job of our consumer is sedentary. In which case, \(x\) represents the fraction of time allocated to exercising and \(1-x\) the fraction of time allocated to work. Consequently, the consumer’s income at \(t\) is:
\[ y(t) = [1 - x(t)]w(t)H(t) = [1 - x(t)]yH(t)t. \]  

With \( p_j(t) \) and \( p_h(t) \) denoting the current prices of junk food and healthy food, respectively, the consumer’s spending on non-food goods at \( t \) is determined by the budget constraint:

\[ s(t) = [1 - x(t)](H(t)t - p_j(t)c_j(t) - p_h(t)c_h(t)). \]  

The consumer derives instantaneous utility from the taste-craving weighted sum of junk food and healthy food, spending on non-food goods and physical activity. For tractability, additive separability is assumed. The marginal instantaneous utilities from \( m \) and \( s \) are taken to be constant, \( \beta_F > 0 \) and \( \beta_{NF} > 0 \). Recalling equation (3), the first part of the latter assumption says that, for simplicity, the marginal instantaneous utility from healthy food is constant, \( \beta_F \), whereas the marginal instantaneous utility from junk food is \( \beta_F \alpha/[1 - A(t)] \). \( A(t) \) changes with the level of junk food consumption. As long as \( c_j \) is smaller (larger) than \( x_c \), craving (aversion) intensifies and the marginal instantaneous utility from junk food increases (diminishes). The second part—constant marginal instantaneous utility from spending on non-food goods—can be justified by arguing that increased spending is associated with a bundle containing larger quantities of luxurious goods. It is assumed that the consumer’s marginal instantaneous pleasure from exercising diminishes and can be negative. In summary, the consumer’s instantaneous utility function is taken to be:

\[ u(m(t), s(t), x(t)) = \beta_F m(t) + \beta_{NF} s(t) + \beta_{x1} x(t) - \beta_{x2} x(t)^2 \]  

where \( \beta_{x1} \) and \( \beta_{x2} \) are positive scalars.

5. Intertemporally bounded rationality and choice

Bounded rationality is a behavioural phenomenon emerging from various reasons and taking various forms (Simon, 1955, 1978). Complexity is a major reason. It also affects the form in which bounded rationality is found (Lee, 2011). The complexity of the system (1)-(8) prevents the computation of the lifetime-utility maximizing joint trajectories of diet and exercising. As in Yaniv et al. (2009), our consumer is concerned about changes in her state but, due to the said complexity, makes her choice in an intertemporally simpler manner. She selects her current diet, consumption of non-food goods and physical activity rate without explicit consideration of future utilities, but with some consideration of the effect of her choice on her state of health and state of craving. With \( \eta > 0 \) and \( \mu > 0 \) indicating the
consumer’s degrees of concern about changes in her health and craving, it is postulated that
the consumer set her current diet and exercising to maximize:

\[ v(t) = \beta_F m(t) + \beta_{NF} s(t) + \beta_{x^1} x(t) - \beta_{x^2} x(t)^2 + \eta \dot{H}(t) - \mu \dot{A}(t). \]  

(9)

This postulated objective function resembles the current-value Hamiltonian of the lifetime-
utility maximization subject to the motion equations of the state variables \( H \) and \( A \). The
parameters \( \eta \) and \( \mu \) can be interpreted as the shadow-prices of health and craving,
respectively, but their evolutions (the co-state equations in the maximum-principle conditions)
are not included in the set of the necessary conditions for maximum \( v(t) \).

Recalling (1)-(7) and omitting the time index for compactness,

\[ v = \beta_F \{[\alpha / (1 - A)]c_j + c_h\} + \beta_{NF} \{(1 - x)\gamma H - p_x c_j - p_h c_h\} + \beta_{x^1} x - \beta_{x^2} x^2 \]

\[ + \eta \{r_b - (1 + r_b)[1 + \delta_x (1 - x) + \delta_h (c_h - (c_{h^{\text{sed}}} + (c_{h^{\text{act}}} - c_{h^{\text{sed}}}))x)^2 + \delta_j c_j^2](t / T_{\text{max}})\} \]

\[ + \mu \theta (1 - A^2)(c_j - x) / \tau_j. \]

(10)

The Hessian of \( v \) is:

\[
D = \begin{bmatrix}
-2\eta(1 + r_b)(t / T_{\text{max}})H\delta_j & 0 \\
0 & -2\eta(1 + r_b)(t / T_{\text{max}})H\delta_h \\
0 & 2\eta(1 + r_b)(t / T_{\text{max}})H\delta_h (c_{h^{\text{act}}} - c_{h^{\text{sed}}}) - [2\beta_{x^2} + 2\eta(1 + r_b)(t / T_{\text{max}})H\delta_h (c_{h^{\text{act}}} - c_{h^{\text{sed}}}^2)]
\end{bmatrix}.
\]

(11)

Since \( D_{11} = -2\eta(1 + r_b)(t / T_{\text{max}})H\delta_j < 0 \), \( D_{11}D_{22} - D_{21}D_{12} = [2\eta(1 + r_b)(t / T_{\text{max}})H]^2\delta_j \delta_h > 0 \) and \( \det D = -8\beta_{x^2}[\eta(1 + r_b)(t / T_{\text{max}})H]^2\delta_j \delta_h < 0 \), the second-order conditions for maximum \( v \)
are satisfied. That is, there exists an interior solution \((c_j^*(t), c_h^*(t), x^*(t))\) to the bounded
rational consumer’s problem.

The first-order conditions for maximum \( v \) are:

\[
\frac{\partial v}{\partial c_j} = \beta_F \alpha / (1 - A) - \beta_{NF} p_j - 2\eta(1 + r_b)(t / T_{\text{max}})H\delta_j c_j^* + \mu \theta (1 - A^2) / \tau_j = 0
\]

(12)

\[
\frac{\partial v}{\partial c_h} = \beta_{NF} p_h - \beta_{NF} p_h - 2\eta(1 + r_b)(t / T_{\text{max}})H\delta_h [c_{h^{\text{act}}} - c_{h^{\text{sed}}} - (c_{h^{\text{act}}} - c_{h^{\text{sed}}})x^*] = 0
\]

(13)

\[
\frac{\partial v}{\partial x} = \beta_{x^1} - 2\beta_{x^2} x^* - \beta_{NF} \gamma H - \eta(1 + r_b)(t / T_{\text{max}})H\delta_x - \mu \theta (1 - A^2)
\]

\[ + 2\eta(1 + r_b)(t / T_{\text{max}})H\delta_h [c_{h^{\text{act}}} - c_{h^{\text{sed}}} - (c_{h^{\text{act}}} - c_{h^{\text{sed}}})x^*](c_{h^{\text{act}}} - c_{h^{\text{sed}}}) = 0.
\]

(14)

From these conditions, the intertemporally bounded rational consumer’s diet and fraction of
time allocated to exercising at \( t \) are:
\[
\begin{align*}
    c_j^*(t) &= \frac{\beta_F \alpha [1 - A(t)] - \beta_{NF} p_j + \mu \theta [1 - A(t)^2]}{2 \eta \delta_j (1 + \eta_j) (t / T_{max}) H(t)} \\
    c_h^*(t) &= [c_h^{sed} + (c_h^{act} - c_h^{sed}) x^*(t)] + \frac{\beta_F - \beta_{NF} p_h}{2 \eta \delta_h (1 + \eta_h) (t / T_{max}) H(t)} \\
    x^*(t) &= \frac{\beta_{x1} + [\eta \delta_x (1 + \eta_x) (1 / T_{max}) - \beta_{NF}] H(t) t - \mu \theta [1 - A(t)^2] + (\beta_F - \beta_{NF} p_h) (c_h^{act} - c_h^{sed})}{2 \beta_{x2}}.
\end{align*}
\]

6. Conclusion

Equation (15) reflects that the bounded rational junk-food consumption at \( t \) is equal to its marginal instantaneous utility plus the value of its marginal contribution to the moderation of craving and minus the forgone instantaneous utility from non-food goods, deflated by the value of its marginal adverse effect on health. As long as \( A(t) < \alpha \bar{c}_j / [2 \mu \theta (1 - A(t))^2] \), the effect of craving on junk-food consumption \( (\partial c_j^*(t) / \partial A) \) is positive. Larger intensities of craving moderate consumption of junk food due to a dominant contribution of that consumption to aversion.

Noting that the denominator of equation (15) is positive, an inspection of the numerator suggests that as long as \( \beta_F [\alpha / (1 - A(t))] + [\mu \theta (1 - A(t)^2)] / \bar{c}_j(t) - \beta_{NF} p_j > 0 \) the bounded rational consumer deviates from the physiologically optimal strategy of abstinence from junk food. The junk-food tax rate that can eliminate her consumption of junk food is \( \beta_F [\alpha / (1 - A(t))] + \mu \theta (1 - A(t)^2) / \bar{c}_j(t) / \beta_{NF} - p_j \), where \( p_j \) is now denoting the price of junk food before tax. This junk-food-consumption eliminating tax rate rises with the consumer’s marginal pleasure from eating \( (\beta_F) \), relative taste for junk food \( (\alpha) \), craving for junk food \( (A) \), degree of concern about craving \( (\mu) \) and digestive system’s sensitivity to junk food \( (\theta) \). The junk-food tax rate decreases with the price of junk food before tax, with the consumer’s marginal pleasure from non-food consumption \( (\beta_{NF}) \) and with digestive discomfort threshold \( (\bar{c}_j) \).

The first term on the right-hand side of equation (16) indicates the consumer’s physiologically optimal consumption of healthy food given her exercising intensity, which is given by equation (17). Equation (16) suggests that the consumption of healthy food is excessive (insufficient) if the marginal current utility from eating healthy food is larger (smaller) than the forgone current utility from consuming non-food goods. The deviation of
healthy-food consumption from the physiologically optimal level is moderated by its adverse effect on health and the consumer’s concern about changes in her health.

Equations (15) and (16) suggest that as the consumer ages the deviation of her healthy food consumption and junk food consumption from her physiologically optimal diet diminishes. Equation (17) suggests that as the consumer ages her deviation from full-time physical activity diminishes (increases) if \( \eta(1 + \eta)(1/T_{\text{max}})\delta_x \) is larger (smaller) than \( \beta_{NF}\gamma \); namely, if the ageing decelerating effect of an additional time spent in exercising is larger (smaller) than the value of the forgone earning. Equation (17) also suggests that the stronger the preference for consuming food, relative to other goods \( (\beta_F - \beta_{NF}p_h) \), the longer the time spent in exercising. That is, an intertemporarily bounded rational person who loves eating counterbalances the ageing accelerating and health deteriorating effect of large food consumption by exercising.
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