Physiological, Gastronomic and Budgetary Aspects and the Diets of Perfectly and Imperfectly Lifetime-Rational Consumers

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This paper analyzes the qualitative and quantitative deviations of rational consumers from their physiologically optimal diets with a distinction between a nutritionally and digestively superior food and a taste and price superior food. The inclusion of a cause-and-effect relationships of these quantitative and qualitative deviations with ageing, craving, digestive discomfort, health-dependent budget, non-food consumption and utility, uncertainty about food’s classification and imperfect dynamic consideration and sophistication adds realistic features to the analysis of rational eating and junk-food tax.

Keywords: Diet; Ageing; Craving; Rationality; Junk-Food Tax
1. Introduction

A difference between a rational choice and the physiologically optimal choice of health-affecting consumption and activities and, consequently, an existence of self-inflicted health problems in a group of rational people have been proposed by Levy (2002a, 2002b). In the context of food-consumption, Levy (2002a) has focused on the quantitative aspect of consumption of a uniform type of food and the consumer’s possible long-run condition. It has led to the proposition that, despite the adverse effect on health and life-expectancy, the steady-state combinations of food-consumption and weight of expected lifetime-utility-maximizing consumers are physiologically excessive and asymptotically unstable. This kind of steady state has been found by Dragone (2009) to be reinforced by habit (loss of utility from inter-temporal changes in the quantity of food-consumption) and approachable from two opposite directions along a singular manifold. Levy’s (2009) re-examination of the asymptotic properties of the physiologically excessive rational steady state has confirmed the existence of a singular stable manifold in the original framework, comprising a converging trajectory of quantity of food consumption and weight from states of underweight and a converging course from more severe states of overweightness. The existence of trajectories of rational convergence to a physiologically excessive steady state provides an explanation to the prevalence of overweight and obesity and their associated diseases and highlights the difficulty in overcoming these health problems.

Well-being is also affected by the qualitative aspects of the food-products constituting the consumers’ diet. Some types of food have opposite effects on the consumers’ instantaneous utility and health: their consumption generates instantaneous utility, but deteriorates health and, subsequently, productivity, income and future utility. The size of the fast-food and snack-food industries suggests that the actual diets of many, not necessarily myopic, consumers deviate significantly from the physiologically optimal strategy of abstinence. Levine et al.’s (2003) study on the neurobiology of preference has shown that central regulatory mechanisms favor foods containing sugar and fat over other nutrients. Having a high concentration of these substances makes physiologically harmful types of food taste-appealing and, possibly, addictive for rational consumers. Furthermore, Philipson and Posner (1999), Lakdawalla and Philipson (2002) and Drenowski (2003) have shown that some food-products are often less expensive than their healthier substitutes due to cheaper ingredients, easier preparation process and storage, and value of time. Using an expected lifetime-utility-maximisation approach, Levy’s (2002c, 2006) studies of the possible implications of taste, price and risk differentials for the consumer’s diet have focused on
steady states and led to the conclusions that the rational stationary junk-food consumption level is equal to the ratio of the recovery capacity of a perfectly healthy person to the health sensitivity to junk food and a tax rate that bridges the gap between the relative market price and the highest relative personal taste of the less healthy food ensures a universal choice of healthy-food diet and leads to the fastest converging path to the highest individual and aggregate levels of health and output. Assuming, implicitly, imperfect lifetime-rationality, Yaniv et al. (2009) have argued that implementation of a fat-tax reduces obesity among non-weight-conscious consumers, but not necessarily among weight-conscious consumers.

The objective of this paper is to combine food-products’ characteristics and consumers’ attributes for constructing a rich model of the divergence of the diets of rational consumers from the physiologically perfect dietary course. The model facilitates the exploration of how the deviations of rational consumers from their physiologically optimal diets are affected by availability of a taste-superior food alongside a nutritionally and digestively superior food, by diet-dependent ageing, by health-dependent income, by utility from other (non-food) goods, and by changes in the consumer’s craving for the taste-superior food. The model further takes into account that the consumer’s craving can be moderated, and even inverted, by recurring episodes of indigestion engendered by flavoring ingredients, fat and lack of fibers.

Furthermore, consumers might not be sure about the classification of some food products, new ones in particular. Also the effectiveness of mandatory labelling in moderating the consumers’ search costs and negative externalities depends on the consumers’ interest in reading labels (Magat and Viscusi, 1992) and reference points (Wuyang et al., 2006). The model is extended to analyze the effect of imperfect information about the qualities of new food-products on the deviations of rational consumers from their physiologically optimal diets.

The proposed conceptualization of the rational choice of the quality and quantity of food refers to the nutritionally and digestively superior food-products as healthy food and to the taste-superior food-products as junk food. The distinction between junk food and healthy food depends on the concentration of calories, fat and flavouring ingredients, whose presence in the human body beyond a critical level is harmful. In addition to a high concentration of these substances, junk food is lacking vital nutrients such as fibres and vitamins.

The conceptualization starts with a deterministic intertemporal model. It is constructed for dynamically sophisticated lifetime-utility maximizing consumers (henceforth, perfectly lifetime-rational consumers) who are able to distinguish between healthy food and
junk food. The components of the deterministic intertemporal model are described in section 2. They include physiological aspects, gastronomic aspects and budgetary aspects of the consumer’s diet. These components are assembled in section 3 to optimally control the consumer’s dietary course over his endogenous lifespan.

As in Yaniv et al. (2009), the case of less farsighted consumers and/or less dynamically sophisticated consumers (henceforth, imperfectly lifetime-rational consumers) is considered in section 4. These consumers maximize their current utility with some consideration of the implications of their current consumption for their condition—the state of their health and the intensity of their craving to junk food, in the present conceptualization.

Inexperience and unclear and/or unreliable labelling make room for uncertainty. Fast-food suppliers and snack-food producers often introduce new tasty brands, which are claimed to be healthy. The introduction of such new brands into a market where the consumers are sceptical is considered in section 5. The consumers’ scepticism is interpreted as uncertainty about the classification of the new brands as healthy and is augmented into the model.

The introduction of the consumers’ craving, ageing, budget, imperfect information and imperfect dynamic consideration and sophistication adds realistic features to the conceptualization and analysis of rational food-consumption. The introduction of ageing eliminates steady states—the focus of the aforementioned earlier studies on rational eating.

2. Physiological, gastronomic and budgetary aspects of diet
Let h denote healthy food and j junk food. Their quantities in the consumer’s diet are measured in units of weight, say grams. For tractability, aggregates of these two general types of food are considered.

2.1 Physiological aspects
Healthy food is physiologically essential. The number of grams of healthy food required for maintaining the consumer in the best possible health at age $t$ is $c_h^0(t) \in \mathbb{R}_+$. Consumption of a larger quantity leads to a loss of health. Junk food is not physiologically essential. The number of grams of junk food required for maintaining the consumer in the best possible health is nil. The combination $(c_h^0(t), 0)$ is the consumer’s physiologically optimal diet at age $t$. It has the highest nutritional value for that particular consumer. Denoting the actual number

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1 With the consumer’s environment and lifestyle taken to be exogenous and time-invariant, $c_h^0$ can be assumed to decline after reaching physiological maturity due to a natural process of decay.
of grams of healthy food and the actual number of grams of junk food consumed at t by $c_h(t) \geq 0$ and $c_j(t) \geq 0$, respectively, the consumer’s actual diet at age t is $(c_h(t), c_j(t))$.

The consumer’s health at any age $t$ is represented by $H(t) \geq 0$. His state of health at birth (initial $H$) is $H_0 > 0$. As long as the consumer adheres to the physiologically optimal diet, $H(t)$ is equal to his best possible health. Due to the inevitable physiological decay there is an (individualistic) upper bound, $T_{\text{max}}$, on the consumer’s life-expectancy ($T$), which can be reached by maintaining the best possible health. Ageing is represented by $t/T_{\text{max}} \in [0,1]$. The consumer perishes when $H$ reaches 0. The consumer’s best possible health changes at a rate that declines from a positive regeneration rate, $r_b$, at birth ($t = 0$) to -1 at the moment of death ($t = T_{\text{max}}$). The terms on the right-hand side of the following health-motion equation display this property. The second term further displays that the adverse effect of ageing on health is intensified by deviations from the optimal diet:

$$\dot{H}(t) / H(t) = r_b - (1 + r_b)[1 + \delta_h(c_h(t) - c_h^0(t))^2 + \delta_j(c_j(t) - 0)^2]t/T_{\text{max}}, \; H_0 > 0.$$  

(1)

The positive scalars $\delta_h$ and $\delta_j$ denote the consumer’s health-sensitivity to deviations of the actual intake from the physiologically optimal quantities of healthy food and junk food, respectively. For simplicity, symmetry is assumed and the possible interaction effect of the two types of deviations is ignored. Equation (1) also displays the following properties. Death (that is, $H/H \leq -1$) is inevitable. For a consumer adhering to the physiologically optimal diet $(c_h^0(t), 0)$, ageing (i.e., $t/T_{\text{max}}$) adversely affects the rate of change of health at a rate $(1 + r_b)$ that exceeds the initial regeneration rate and hence health peaks at $t = [r_b / (1 + r_b)]T_{\text{max}}$. Thereafter, ageing dominates regeneration and, consequently, health deteriorates and is completely eroded at $T_{\text{max}}$ (that is, $\dot{H}(T_{\text{max}})/H(T_{\text{max}}) = -1$).

### 2.2 Gastronomic aspects

While the consumer’s health is affected by the nutritional value of his diet, the consumer’s pleasure from diet is determined by the taste and digestive comfort of his diet. Due to a high concentration of flavouring substances, junk food is tastier for the consumer than its healthier substitute. This property is expressed by letting the taste of healthy food be equal to 1 and the (relative) taste of junk food be indicated by $\alpha > 1$. However, beyond a critical level of junk-food consumption, $\bar{c}_j \geq 0$, the overdose of the flavoured, fat-rich and fibre-poor food causes
nausea, heartburn, upset-stomach and/or constipation. The scalar $\overline{c}_j$ can be interpreted as the consumer’s digestive discomfort threshold. The larger the overdose $(c_j - \overline{c}_j)$ is, the stronger the consumer’s digestive discomfort. The digestive discomfort experienced at time $t$ intensifies the consumer’s aversion to junk food and thereby moderates its future consumption. Hence, it is possible that the present junk-food consumption of a consumer with a strong relative taste for junk food, but a sensitive digestive system, is moderated significantly by past overdosing. In contrast, when the consumption of junk food is smaller than $\overline{c}_j$, the digestive discomfort-free taste intensifies the consumer’s state of craving to junk food due to addiction to its flavouring ingredients. It is therefore possible that the present junk-food intake of a consumer with a weak relative taste for junk food is increased by moderate past consumption. Due to digestive comfort, or discomfort, the consumer’s current attraction ($-1 \leq A(t) \leq 1$) to junk food evolves from an initial state of unfamiliarity-based indifference ($A(0) = 0$) to a state of craving ($0 < A \leq 1$), or aversion ($-1 \leq A < 0$), interchangeably. Aversion diminishes the consumer’s pleasure from eating junk food, whereas craving intensifies. With this argument in mind, the absolute value of the change in the consumer’s state of aversion (craving) to junk food is assumed to rise with overdosing (under-dosing), but in a rate that diminishes with the already existing intensity:

$$\dot{A}(t) = -\theta [1 - A(t)^2] \left[\frac{[c_j(t) - \overline{c}_j(t)]}{\overline{c}_j(t)}\right].$$

(2)

The scalar $0 < \theta \leq 1$ reflects the sensitivity of the consumer’s digestive system to junk food. Starting life with unfamiliarity-based indifference to junk food ($A(0) = 0$), equation (2) ensures that $-1 \leq A(t) \leq 1$ for every $t \in [0, T^*]$.

The consumer’s pleasure from his diet at age $t$ is represented by a function $u^F(c_h(t), c_j(t))$ that has the following properties. Neither healthy food nor junk food is gastronomically essential: $u^F(0, c_j), u^F(c_h, 0) > 0$. The marginal instantaneous pleasure with respect to each type of food is positive but diminishing: $u^F_{c_h}, u^F_{c_j} > 0$ and $u^F_{c_h c_h}, u^F_{c_j c_j} < 0$. The ratio of the marginal pleasures from junk food and healthy food increases with the intrinsic relative taste of junk food, but is diminished by a rise in the consumer’s aversion to junk food. With these assumptions in mind, the sum of the quantities of junk food and healthy food consumed

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2 The consumer’s digestive discomfort threshold can rise initially with age and then decline due to the natural process of physiological growth and decay.
at t and weighted by taste and craving (or aversion), m, is introduced as the argument of a non-convex function displaying the consumer’s pleasure from eating. That is,

\[ u^F(t) = u^F(m(t)) \]  \hspace{1cm} (3)

where

\[ m(t) = \left( \frac{\alpha}{1 - A(t)} \right) c_j(t) + c_h(t) \]. \hspace{1cm} (4)

The ratio \( \alpha / [1 - A(t)] \) indicates the consumer’s relative marginal pleasure from the junk food component of his diet at age t (i.e., \( u_j^F / u_h^F = \alpha / [1 - A(t)] \)). If, for example, by age t the consumer has developed some aversion to junk-food (\( -1 \leq A(t) < 0 \)) through past episodes of overdosing discomfort, his relative marginal pleasure from junk food at t is smaller than the relative taste of junk food (\( \alpha \)).

### 2.3 Budgetary aspects

Health affects the consumer’s income and, in turn, budget. Recalling equation (1), the consumer’s budget is indirectly affected by his past and present diets through their effects on his health. Knowledge and experience determine the current rate of return, \( w(t) \), on the consumer’s health. Since knowledge and experience are accumulated over time, \( w \) is taken to be growing over the lifespan, for simplicity, at a constant rate \( \gamma \):

\[ w(t) = \gamma t. \] \hspace{1cm} (5)

In order to simplify the ensuing sections’ analyses the issues of the time allocated to the preparation of the healthy food and the forgone income are avoided by assuming that a market for healthy food exists, which is the common case in technologically advanced countries. Consequently, the consumer’s income at t is:

\[ y(t) = w(t)H(t) = \gamma H(t)t \]. \hspace{1cm} (6)

The positive scalar \( \gamma \) can also be interpreted as the return on a health-adjusted moment of experience. This specification, in conjunction with the health-motion equation (1), suggests an inverted U-shaped income curve over the lifecycle with zero initial income (due to having no knowledge and experience at \( t=0 \)) and zero terminal income (due to \( H(T)=0 \)).

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3 If a market for healthy food did not exist and \( 0 < \tau_h(t) < 1 \) were the consumer’s current preparation time per gram of healthy food, the consumer would have had to allocate \( \tau_h(t)c_h(t) \) for preparing healthy food and, consequently, earned \( \gamma tH(t)[1 - \tau_h(t)c_h(t)] \) at t.
With \( p_j(t) \) and \( p_h(t) \) denoting the current market prices of junk food and healthy food, respectively, the consumer’s spending on non-food goods at \( t \) is determined by his budget constraint:

\[
s(t) = \gamma H(t)t - p_j(t)c_j(t) - p_h(t)c_h(t).
\]

For simplicity, the consumer’s instantaneous pleasure from the consumption of non-food goods is taken to be independent of his diet and given by a monotonically increasing non-convex function:

\[
u_{NF}^{NF}(t) = u_{NF}(s(t)).
\]

3. Perfectly lifetime-rational consumer’s choice of diet

The consumer’s instantaneous pleasure from eating and instantaneous pleasure from consuming non-food goods constitute the consumer’s instantaneous utility:

\[
u(t) = u^F(m(t)) + u_{NF}(s(t)).
\]

A lifetime-rational consumer chooses a diet trajectory at \( t_0 < T^* \) that maximizes his lifetime utility, \( U(u_0, ..., u_{t_0}) \), subject to the health and attraction motion-equations (1) and (2). For simplicity, the lifetime-utility function of that sophisticated, self-controlled consumer is additively separable in the instantaneous utilities and his time-preferences are consistent and represented by a positive time-invariant rate \( \rho \). As indicated earlier, a physiologically non-optimal diet prevents the consumer from living up to his utmost life-expectancy \( T \). He perishes at \( T^* \leq T \). Prior to choosing his diet-path, \( T^* \) is not yet determined. The realization of \( H(t) = 0 \) and, consequently, \( y(t) = 0 \) and \( u(t) = 0 \) at any \( t \in (T^*, T) \) permits integration of the discounted instantaneous utilities over the longest possible planning horizon:

\[
U \equiv \int_{t_0}^{T^*} e^{-\rho t}u(t)dt = \int_{t_0}^{T} e^{-\rho t}u(t)dt.
\]

By substituting the information embedded in equations (9), (7), and (4) into (10) the lifetime-rational consumer’s decision problem is portrayed as choosing the diet course \( \{c_h, c_j\} \) that maximizes \( \int_{t_0}^{T} e^{-\rho t}\{u^F(\frac{\alpha}{(1-A)}c_j + c_h) + u_{NF}(\gamma H(t) - p_jc_j - p_hc_h)\}dt \) subject to the motion-equations (1) and (2) of health and craving. With the time-index omitted for compactness, the present-value Hamiltonian associated with this optimal-control problem is:
\[ \mathcal{H} = e^{-\rho t} \left\{ u^F(\alpha / (1 - A) | c_j + c_h) + u^{NF}(\gamma_H t - p_j c_j - p_h c_h) \right\} + \lambda_H \{ r_h - (1 + r_h)(1 + \delta_h(c_h - c_h^0)^2 + \delta_j c_j^2)(t / T) \} H + \lambda_A (A^2 - 1)(c_j - \overline{c}_j) / \overline{c}_j. \]  

(11)

The co-state variable \( \lambda_H \) indicates the present-value shadow price of health for the consumer, and the co-state variable \( \lambda_A \) the present-value shadow price of the consumer’s state of craving to junk food. In addition to the instantaneous utility from consuming food and other goods the Hamiltonian includes the value of the changes in the consumer’s states of health and aversion to junk food. While \( \lambda_H \) is positive, the sign of \( \lambda_A \) is not clear. On the one hand, a slight intensification of the consumer’s craving increases junk-food consumption and, consequently, decreases his future health and, consequently, future incomes and utilities and life-expectancy. On the other hand, a rise in craving increases the consumer’s instantaneous pleasure from junk food. Hence, \( \lambda_A \) is negative (positive) when the adverse effect of the loss of health on the consumer’s lifetime utility is larger (smaller) than the positive effect of the enhanced pleasure from consuming junk food. The intertemporal changes in the shadow prices of the rational consumer’s states of health and craving to junk food are equal to the effect of a slight decline of these states on the value of the Hamiltonian:

\[ \hat{\lambda}_H = -\frac{\partial \mathcal{H}}{\partial H} = -e^{-\rho t} u_s^{NF}(s)\gamma_H t - \lambda_H \{ r_h - (1 + r_h)(1 + \delta_h(c_h - c_h^0)^2 + \delta_j c_j^2)(t / T) \} \]  

(12)

\[ \hat{\lambda}_A = -\frac{\partial \mathcal{H}}{\partial A} = -e^{-\rho t} u_m^F(m) \frac{\alpha c_j}{(1 - A)^2} - 2\lambda_A (A^2 - 1)(c_j - \overline{c}_j) / \overline{c}_j \]  

(13)

The convexity of the loss of health from deviations from the physiologically optimal diet and the concavity of \( u^F \) and \( u^{NF} \) ensures that the Hamiltonian is concave in the control variables. In addition to the shadow-prices’ (adjoint) equations (12) and (13) and the health and craving state-equations (1) and (6), the set of the necessary conditions for maximum includes:

\[ \frac{\partial \mathcal{H}}{\partial c_j} = e^{-\rho t} \left\{ u_m^F(m)[\alpha / (1 - A)] - u_s^{NF}(s)p_j \right\} - 2\lambda_H \delta_j (1 + r_h)(t / T)c_j H + \lambda_A (A^2 - 1) / \overline{c}_j = 0 \]  

(14)

\[ \frac{\partial \mathcal{H}}{\partial c_h} = e^{-\rho t} \left\{ u_m^F(m) - u_s^{NF}(s)p_h \right\} - 2\lambda_H \delta_h (1 + r_h)(t / T)(c_h - c_h^0) H = 0 \]  

(15)

\[ \lambda_H(T)H(T) = 0 \]  

(16)

\[ \lambda_A(T)A(T) = 0. \]  

(17)
The optimality conditions (14) and (15) imply that along the perfectly lifetime-rational diet-path the net marginal pleasure is equal to the marginal damage to health generated by excessive eating of either type of food, and the ratio of the net marginal pleasures is equal to the ratio of the marginal damages to health and ability to enjoy junk food:

\[ \frac{u^F_m(m)[\alpha/(1-A)]-u^\text{NF}_s(s)p_j}{u^F_m(m)-u^\text{NF}_s(s)p_h} = \frac{2\lambda_h\delta_j(1+r_b)(t/T)\alpha H - \lambda_A\theta(A^2-1)/\bar{e}_j}{2\lambda_h\delta_h(1+r_b)(t/T)(c_h-c_h^0)H}. \]  

(18)

The optimality condition (14) also implies that \( \frac{u^F_m(m)[\alpha/(1-A)]-u^\text{NF}_s(s)p_j}{u^F_m(m)-u^\text{NF}_s(s)p_h} > 0 \) as long as

\[ \frac{\lambda_h}{\lambda_A} > \frac{2\delta_j(1+r_b)(t/T)\alpha H}{\{[\theta(A^2-1)]/\bar{e}_j\}}. \]

That is, the marginal pleasure from eating junk food is larger (smaller) than the forgone pleasure from consuming non-food goods as long as the ratio of the shadow values of craving and health is smaller (larger) than the ratio of the marginal rate of decline in the consumer’s health and the marginal rise of his aversion engendered by junk-food consumption. From (15), \([u^F_m(m)-u^\text{NF}_s(s)p_h] > 0 \) as long as

\[ (c_h-c_h^0) > 0. \]

That is, the marginal pleasure from physiologically excessive (insufficient) eating of healthy food must be larger (smaller) than the forgone pleasure from consuming non-food goods.

As detailed in Appendix A, the differentiation of the optimality conditions with respect to time and the substitution of the shadow prices and state equations lead to the Euler conditions of junk-food and healthy-food consumption. For tractability, let us consider the case where \( u^F_m \) and \( u^\text{NF}_m \) are linear (\( u^F_m = u^\text{NF}_m \)) and the prices of junk food and healthy food are time-invariant. In this case, the convexity of the health-rate loss function in the deviations from the physiologically optimal diet ensures that the Hamiltonian is still concave in the control variables and the associated Euler conditions are:

\[ \dot{c}_j = \left[ \frac{(c_h-c_h^0)c_j}{[u^F_m-u^\text{NF}_s(p_h)]} \left( \frac{\delta_h}{\delta_j}c_j \right)u^F_m(\alpha \theta + 2(1+r_b)(t/T)\alpha H u^\text{NF}_s\gamma t \right. \right. \\
\left. \left. \left[ (\delta_h/\delta_j)c_j \right] \{u^F_m(\alpha/(1-A))-u^\text{NF}_s(p_j)\rho - (\delta_h/\delta_j)(t/t)\} \right]\right]. \]  

(19)

and

\[ \frac{d(c_h-c_h^0)}{dt} = 2\delta_h(1+r_b)(t/T)(c_h-c_h^0)^2 Hu^\text{NF}_s(s)\gamma t \right. \right. \\
\left. \left. \left[ u^F_m-u^\text{NF}_s(p_h) \right\] - (c_h-c_h^0)(\rho + 1/t). \]  

(20)
These Euler conditions indicate that if, and only if, it were optimal at \( t \) to have 
\( c_h(t) = c_h^0(t) \), then the consumption of junk food should remain unchanged and the change in 
the consumption of healthy food should match the change in its physiologically optimal level. 
From equation (20), the effect of the consumer’s time-preference rate on the change in the 
consumption of healthy food depends on whether the current consumption of healthy food is 
excessive or insufficient: 
\[
\frac{\partial \hat{c}_h}{\partial \rho} \quad \text{as} \quad (c_h - c_h^0) > 0. 
\]
Equation (20) indicates further that as long 
as the marginal utility from health food exceeds the forgone utility from non-food (i.e., 
\( u_{m}^F - u_{s}^N p_h > 0 \)), the consumption of healthy food by a perfectly lifetime-rational consumer 
(though with constant marginal utilities from food and non-food) rises with his age (\( t \)), with 
his wage-age gradient (\( \gamma \)), and with his natural rate of physiological decay (1 + \( r_h \)) 
compounded by his sensitivity to deviations from the physiologically optimal health-food 
intake (\( \delta_h \)).

4. Imperfectly lifetime-rational consumer’s choice of diet

Let us now consider the case of a less farsighted and/or sophisticated consumer who derives 
satisfaction from consuming food and other goods without explicit consideration of future 
utilities, but with some concerns about the deterioration of his health and intensification of 
his craving to junk food. At every \( t \) this imperfectly lifetime-rational consumer chooses the 
instantaneous diet \( (c_h(t), c_j(t)) \) that maximises his current overall utility from food, from 
other goods and from changes in his health and craving. With \( \eta > 0 \) and \( \mu > 0 \) indicating the 
degrees of his concern about the deterioration of his physiological and mental conditions, the 
imperfectly lifetime-rational consumer’s current overall utility is:

\[
v(t) = u^F(m(t)) + u^N(s(t)) + \eta \hat{H}(t) - \mu \hat{A}(t). 
\]

Recalling (1)-(8),

\[
v(t) = u^F(\{\alpha / (1 - A(t))\}c_j(t) + c_h(t)) + u^N(\gamma H(t)t - p_j c_j(t) - p_h c_h(t)) 
\]
\[
+ \eta(t_h - (1 + r_h))[1 + \delta_h (c_h(t) - c_h^0(t))^2 + \delta_j c_j(t)^2](t / T)H(t) 
\]
\[
+ \mu \theta(1 - A(t)^2)(c_j(t) - \overline{c}_j(t))/\overline{c}_j(t). 
\]

Assuming, for tractability, that the marginal pleasures from food and other goods are constant 
and equal to \( \beta_f > 0 \) and \( \beta_{nf} > 0 \), the imperfectly lifetime-rational consumer’s chosen diet at \( t \) 
includes:
\[ c^*_j(t) = \frac{\beta_F[\alpha/(1-A(t))] + [\mu\theta(1-A(t)^2)/\overline{z}(t)] - \beta_{NF}p_j}{2\eta(1+n_h)_j\delta_j(t/T)H(t)} \]  

and

\[ c^*_h(t) - c^*_n(t) = \frac{\beta_F - \beta_{NF}p_h}{2\eta(1+n_h)_j\delta_h(t/T)H(t)}. \]  

Equation (23) reflects that the imperfectly lifetime-rational consumer’s junk-food consumption at \( t \) is equal to its marginal utility plus the value of its marginal contribution to the moderation of craving and minus the forgone current utility from other goods, deflated by the value of its marginal adverse effect on health. As long as \( \beta_F[\alpha/(1-A(t))] + [\mu\theta(1-A(t)^2)/\overline{z}(t)] - \beta_{NF}p_j > 0 \), the effect of craving on junk-food consumption \( (\partial c^*_j(t)/\partial A) \) is positive. Larger intensities of craving moderate consumption of junk food due to a dominant contribution of that consumption to aversion.

Noting that the denominator of Equation (23) is positive, an inspection of the numerator suggests that as long as \( \beta_F[\alpha/(1-A(t))] + [\mu\theta(1-A(t)^2)/\overline{z}(t)] - \beta_{NF}p_j > 0 \) the imperfectly lifetime-rational consumer deviates from the physiologically optimal strategy of abstinence from junk food. In which case the junk-food tax rate that eliminates his consumption of junk food is:

\[ \tau_j = \frac{\beta_F[\alpha/(1-A(t))] + [\mu\theta(1-A(t)^2)/\overline{z}(t)] - \beta_{NF}p_j}{\beta_{NF}} p_j \]  

where \( p_j \) is now denoting the pre-tax price of junk food. The junk-food tax rate increases with the consumer’s marginal pleasure from eating \( (\beta_F) \), relative taste for junk food \( (\alpha) \), craving \( (A) \), degree of concern about craving \( (\mu) \) and digestive system’s sensitivity to junk food \( (\theta) \). The junk-food tax rate decreases with the pre-tax price of junk food and with the consumer’s marginal pleasure from non-food consumption \( (\beta_{NF}) \) and digestive discomfort threshold \( (\overline{z}) \).

Equation (24) suggests that the consumption of healthy food is excessive (insufficient) if the marginal current utility from eating healthy food is larger (smaller) than the forgone current utility from consuming non-food goods. The deviation of healthy-food consumption from the physiologically optimal level is moderated by its adverse effect on health and the consumer’s concern about a change in his health.
5. Choice of diet with imperfect information

Experience eliminates consumers’ uncertainty about the qualities of old brands. The deterministic model is extended in this section to investigate the effects of uncertainty about new food-products classification on the consumer’s diet. New brands of fast food and snack food are introduced and claimed by their suppliers to be healthy. Hence, the equally tasty new brands are usually priced higher than the older ones. In the absence of clear labelling, the consumers are sceptical about the suppliers’ claims. From the perspective of the consumers, the new brands constitute a third type of food, new food (n) priced \( p_n > p_j \), with a probability \( 0 < \psi < 1 \) that a fraction \( 0 < \epsilon \leq 1 \) of its consumption \( c_n \geq 0 \) is junk. The more sceptical the consumers are the larger \( \psi \) and \( \epsilon \). Consequently, the effective quantity of junk food \( c_j^e \) and the effective quantity of healthy food \( c_h^e \) are perceived by a consumer of the new brands to be random variables with the following binomial distributions:

\[
c_j^e(t) = \begin{bmatrix} (c_j(t) + \epsilon c_n(t)) \psi \\ c_j(t) \end{bmatrix} 1 - \psi \tag{26}
\]

and

\[
c_h^e(t) = \begin{bmatrix} (c_h(t) + (1-\epsilon)c_n(t)) \psi \\ [c_h(t) + c_n(t)] \end{bmatrix} 1 - \psi. \tag{27}
\]

In turn, the changes in the consumer’s health and attraction to junk food are also random:

\[
\bar{H} = \begin{bmatrix} \{\eta_h - (1 + \eta_h)\{(c_h + \epsilon c_n) - c_h^0\}^2 + \delta_j (c_j + \epsilon c_n)^2\} (t / T_{\text{max}}) \}H \psi \\
\{\eta_h - (1 + \eta_h)\{1 + \delta_h ((c_h + c_n) - c_h^0)^2 + \delta_j c_j^2\} (t / T_{\text{max}}) \}H \psi \end{bmatrix} 1 - \psi \tag{28}
\]

and

\[
\hat{A} = \begin{bmatrix} -0[1 - \Lambda^2] (c_j + \epsilon c_n - \overline{c}_j) / \overline{c}_j \psi \\
-0[1 - \Lambda^2] (c_j - \overline{c}_j) / \overline{c}_j \psi \end{bmatrix} 1 - \psi. \tag{29}
\]

It is impossible to construct an expected lifetime utility and an optimal control problem with (26)-(29). Hence, only the diet of an imperfectly lifetime-rational consumer is analyzed.

Facing uncertainty, the imperfectly lifetime-rational consumer is taken to be maximizing expected current overall utility which, in recalling (21), is:

\[
Ev(t) = \beta_E E(m(t)) + \beta_{NF} E(s(t)) + \eta E(\bar{H}(t)) - \mu E(\hat{A}(t)). \tag{30}
\]

As shown in Appendix C, the consumer’s chosen diet at \( t \) includes:
\[ c_n^* = \frac{\beta_F(\alpha-1)(1-\epsilon\psi) + \beta_{NF}(\epsilon\psi\rho + (1-\psi\epsilon)p_h - p_n) - (1-\psi\epsilon)\mu[1-A^2]/c_j}{2\eta(1+r_h)(t/T_{max})H(\delta_h+\delta)\psi(1-\psi)\epsilon^2} \]  

(31)

\[ c_j^* = \frac{\beta_F(\alpha/1-A) + \mu[1-A^2]/c_j - \beta_{NF}\rho_p}{2\eta(1+r_c)(t/T_{max})H\delta_j} - \psi\epsilon c_n^* \]  

(32)

\[ c_h^* = c_h^0 + \frac{\beta_F + \mu[1-A^2]/c_j - \beta_{NF}p_h}{2\eta(1+r_h)(t/T_{max})H\delta_h} - (1-\psi)\epsilon c_n^*. \]  

(33)

As long as \( \psi \) and \( \epsilon \) are not both equal to 1, there is some consumption of the new brands at a price higher than \( p_j \). If \( \psi \) and \( \epsilon \) are sufficiently small, the suppliers can even expect consumption of the new brands with \( p_n \) higher than \( p_h \). In view of the budget constraint, the consumption of the new brands lowers the consumption of the clearly recognized junk food and healthy food. An inspection of equation (31) reveals that the consumption of the new brands is moderated by ageing and the consumer’s concern about his health. It is also moderated by the price of these brands, sensitivity of his digestive system and his concern about craving to (the identically tasty) junk food. As long as healthy food is taste-inferior, the consumption of the equally tasty new brands rises with the consumer’s marginal utility from eating. The consumption of the new brand is moderated by the marginal utility from non-food consumption if, and only if, the price of the new brands exceeds the sum of the prices of the clearly recognised junk food and healthy food weighted by the expected shares of junk and healthy components in the new brands (i.e., \( \frac{\partial c_n^*}{\partial\beta_{NF}} < 0 \) as long as \( p_n > \psi\epsilon\rho + (1-\psi\epsilon)p_h \)). Equation (32) suggests that the consumption of the clearly recognized junk food is moderated by the share of the consumption of the new brands expected to be junk. Similarly, equation (33) implies that the consumption of the clearly recognized healthy food is moderated by the share of the consumption of the new brands expected to be healthy.

6. Summary

Rational food consumption deviates from the physiologically optimal diet that requires moderate consumption of calories and abstinence from food-products containing excessive quantities of fat and salt. The objective of this paper was to add realistic features to the analysis of the cause and effect of this deviation.
The first feature was ageing. This process occurs naturally, but it can be accelerated by the deviation of the consumer from the physiologically optimal diet. The incorporation of ageing eliminates steady states—the focus of the earlier studies on rational food-consumption indicated in the introduction.

The second feature was craving. It provided a rationale for increasing deviations from the physiologically optimal diet. As long as the consumption is below the consumer’s digestive discomfort threshold, his craving to junk food intensifies with the consumption of junk food. When junk-food consumption exceeds the digestive discomfort threshold, craving is moderated. Repeated episodes of digestive discomfort can develop aversion to junk food.

The third feature was health-dependent income and budget, which implies that the consumer’s consumption of healthy food, junk food and other goods depends on his past and present diets.

The fourth feature was imperfect dynamic consideration and/or sophistication. Instead of lifetime utility, the consumer maximises his current overall utility which, in addition to current utility from consumption, takes into account the effects of his current consumption on his health and craving conditions.

The fifth feature was imperfect information about new food-products and, consequently, consumers’ uncertainty about the contents of junk food and healthy food in their diet.

As detailed in sections 3, 4 and 5, the incorporation of these features enriched the analysis of the quantity and composition of rational food consumption, their deviation from the physiologically optimal diet and the tax rate that eliminates junk-food consumption.
APPENDIX A: The Euler conditions for a lifetime rational consumer

A.1 Euler condition of junk-food consumption

By differentiating (14) with respect to time,

\[-pe^{-pt}(u^F_m(m)\alpha/(1-A) - u^S_m(s)p_j) + e^{-pt}u^m_{mm}(m)\alpha/(1-A)\dot{m} + e^{-pt}u^F_m(m)\alpha/(1-A)^2\dot{A} - e^{-pt}u^S_m(s)p_j\dot{s} - e^{-pt}u^S_m(s)\dot{p}_j \]

\[-2\lambda_{it}\delta_j(1 + r_b)(t/T)c_jH - 2\lambda_{it}\delta_j(1 + r_b)(t/T)H\dot{c}_j - 2\delta_j(1 + r_b)(t/T)c_jH\dot{\lambda}_{H}\]

\[-2\lambda_{it}\delta_j(1 + r_b)(t/T)c_jH + (\theta/\bar{c}_j)\dot{\lambda}_A(A^2 - 1) + 2\lambda_A(\theta/\bar{c}_j)\dot{A}A = 0 \]

By substituting (12) and (13),

\[-pe^{-pt}(u^F_m(m)\alpha/(1-A) - u^S_m(s)p_j) + e^{-pt}u^m_{mm}(m)\alpha/(1-A)\dot{m} + e^{-pt}u^F_m(m)\alpha/(1-A)^2\dot{A} - e^{-pt}u^S_m(s)p_j\dot{s} - e^{-pt}u^S_m(s)\dot{p}_j \]

\[-2\lambda_{it}\delta_j(1 + r_b)(t/T)c_jH + 2\lambda_A(\theta/\bar{c}_j)\dot{A}A \]

\[-(\theta/\bar{c}_j)(A^2 - 1)[e^{-pt}u^F_m(m)\alpha c_j/(1-A)^2 + 2\lambda_A(\theta/c_j - \bar{c}_j)/\bar{c}_j] = 0 \]

Recalling (6),

\[-pe^{-pt}(u^F_m(m)\alpha/(1-A) - u^S_m(s)p_j) + e^{-pt}u^m_{mm}(m)\alpha/(1-A)\dot{m} + e^{-pt}u^F_m(m)\alpha\theta - e^{-pt}u^S_m(s)p_j\dot{s} - e^{-pt}u^S_m(s)\dot{p}_j \]

\[-2\lambda_{it}(1 + r_b)(t/T)H\delta_j\dot{c}_j + 2(1 + r_b)(t/T)H\delta_jc_je^{-pt}u^S_m(s)\gamma t - 2\lambda_{it}(1 + r_b)(1/T)H\delta_jc_j = 0 \]

From (15),

\[2\lambda_{it}(1 + r_b)(t/T)H = e^{-pt}[u^F_m(m) - u^S_m(s)p_h]/[\delta_h(c_h - c^0_h)] \]

Hence,

\[-p(u^F_m(m)\alpha/(1-A) - u^S_m(s)p_j) + e^F_m(m)\alpha\theta - u^S_m(s)p_j\dot{s} - u^S_m(s)\dot{p}_j \]

\[-(\delta_j/\delta_h)[u^F_m(m) - u^S_m(s)p_h]/(c_h - c^0_h)][\dot{c}_j + c_j/t] + 2(1 + r_b)(t/T)H\delta_jc_ju^S_m(s)\gamma t = 0 \]

In the special case where \(u^F\) and \(u^S\) are linear and the prices of junk-food and healthy-food are time-invariant,

\[-p(u^F_m[\alpha/(1-A)] - u^S_m(s)p_j) + e^F_m\alpha\theta - (\delta_j/\delta_h)[u^F_m - u^S_m(s)p_h]/(c_h - c^0_h)][\dot{c}_j + c_j/t] + 2(1 + r_b)(t/T)H\delta_jc_ju^S_m(s)\gamma t = 0 \]

and the Euler equation (19) is obtained by rearranging terms.
A.2 Euler condition of healthy-food consumption

By differentiating (15) with respect to time,

\[
-\rho e^{-\rho t}[u^F(m) - u^NF(s)p_h] + e^{-\rho t}u^F_{mm}(m)m - e^{-\rho t}u^NF_{ss}(s)p_h \dot{s} - e^{-\rho t}u^NF(s)\dot{p}_h
-2\lambda_\mu \delta_h(1 + \eta)(t/T)(c_h - c^o_h)\dot{H} - 2\delta_h(1 + \eta)(t/T)(c_h - c^o_h)\dot{H} = 0
\]

(A7)

Recalling (12),

\[
-\rho e^{-\rho t}[u^F(m) - u^NF(s)p_h] + e^{-\rho t}u^F_{mm}(m)m - e^{-\rho t}u^NF_{ss}(s)p_h \dot{s} - e^{-\rho t}u^NF(s)\dot{p}_h
+2\delta_h(1 + r_b)(t/T)(c_h - c^o_h)H[e^{-\rho t}u^F_s(s)\gamma t]
-2\lambda_\mu \delta_h(1 + \eta)(t/T)H[(c_h - c^o_h) + (c_h - c^o_h)/t] = 0
\]

(A8)

Recalling (A4),

\[
-\rho[u^F(m) - u^NF(s)p_h] + u^F_{mm}(m)m - u^NF_{ss}(s)p_h \dot{s} - u^NF(s)\dot{p}_h
+2\delta_h(1 + r_b)(t/T)(c_h - c^o_h)Hu^F_{ss}(s)\gamma t
-\{(u^F(m) - u^NF(s)p_h)/((c_h - c^o_h))[[\dot{c}_h - \dot{c}_o_h] + (c_h - c^o_h)/t] = 0
\]

(A9)

In the special case where $u^F$ and $u^NF$ are linear and the prices of junk-food and healthy-food are time-invariant,

\[
-\rho[u^F(m) - u^NF(s)p_h] + 2\delta_h(1 + r_b)(t/T)(c_h - c^o_h)Hu^F_{ss}(s)\gamma t
-\{(u^F(m) - u^NF(s)p_h)/((c_h - c^o_h))[[\dot{c}_h - \dot{c}_o_h] + (c_h - c^o_h)/t] = 0
\]

(A10)

and the Euler equation (20) is obtained by rearranging terms.

APPENDIX B: Imperfectly lifetime-rational consumer’s choice

With constant marginal pleasure from food and other good, $\beta_F > 0$ and $\beta_{NF} > 0$,

\[
v = \beta_F \{[\alpha/(1-A)]c_j + c_h\} + \beta_{NF}(\gamma Ht - p_jc_j - p_hc_h)
+ \eta\{r_b - (1 + r_b)[1 + \delta_h(c_h - c^o_h)^2 + \delta_jc_j^2](t/T)\}H
+ \mu\theta(1-A^2)(c_j - \overline{c}_j)/\overline{c}_j
\]

(B1)

The necessary condition for maximum (with the time index omitted for compactness) are:

\[
\partial v/\partial c_j = \beta_F[\alpha/(1-A)] - \beta_{NF}p_j - 2\eta(1 + r_b)\delta_j(t/T)Hc_j^* + \mu\theta(1-A^2)/\overline{c}_j = 0
\]

(B2)

\[
\partial v/\partial c_h = \beta_F - \beta_{NF}p_h - 2\eta(1 + r_b)\delta_h(t/T)H(c_h^* - c^o_h) = 0
\]

(B3)

As

\[
\partial^2 v/\partial c_j^2 = -2\eta(1 + r_b)\delta_j(t/T)H < 0
\]

(B4)

\[
\partial^2 v/\partial c_h^2 = -2\eta(1 + r_b)\delta_h(t/T)H < 0
\]

(B5)
\[(\partial^2 v / \partial c_j^2)(\partial^2 v / \partial c_h^2) - (\partial^2 v / \partial c_j \partial c_h)^2 = [2\eta(1 + r_h)(t / T)H]^2 \delta_j \delta_h > 0 \quad (B6)\]

the second-order conditions for maximum are satisfied and the interior solution indicated by (23) and (24) are obtained from (B2) and (B3).

**APPENDIX C: Choice with imperfect information**

By substituting (8), (4), (1) and (6) into (30),

\[
E(v) = \psi \beta F[(\alpha / (1 - A))(c_j + \epsilon c_n) + \alpha(1 - \epsilon)c_n + c_h] + (1 - \psi)\beta F[(\alpha / (1 - A))c_j + \alpha c_n + c_h] + \beta_{NF}(\gamma Ht - p_j c_j - p_h c_h - p_n c_n) + \psi \eta \{r_b - (1 + r_b)\text{max }(1 + \delta_h((c_h + (1 - \epsilon)c_n) - c_h^o)^2 + \delta_j(c_j + \epsilon c_n)^2\}]H + (1 - \psi)\eta \{r_b - (1 + r_b)\text{max }(1 + \delta_h((c_h + c_n) - c_h^o)^2 + \delta_j c_j^2\}]H + \psi \mu \theta[1 - A^2][c_j + \epsilon c_n] / \bar{c}_j + (1 - \psi)\mu \theta[1 - A^2][c_j - \bar{c}_j] / \bar{c}_j
\]

(C1)

The necessary conditions for maximum expected current overall utility are:

\[
\frac{\partial E(v)}{\partial c_j} = \beta F[(\alpha / (1 - A)] - \beta_{NF} p_j - 2\psi \eta(1 + r_b)(t / T_{max})H \delta_j \epsilon c_n^* - 2\eta(1 + r_b)(t / T_{max})H \delta_j c_n^* + \mu \theta[1 - A^2] / \bar{c}_j = 0 \quad (C2)
\]

\[
\frac{\partial E(v)}{\partial c_h} = \beta F - \beta_{NF} p_h + \mu \theta[1 - A^2] / \bar{c}_j - 2\eta(1 + r_b)(t / T_{max})H \delta_h (1 - \epsilon \psi) c_n^* - 2\eta(1 + r_b)(t / T_{max})H \delta_h c_h^o = 0 \quad (C3)
\]

\[
\frac{\partial E(v)}{\partial c_n} = \psi \beta F[(\alpha / (1 - A))\epsilon + (1 - \epsilon)] + (1 - \psi)\beta F \alpha - \beta_{NF} p_n - 2\psi \eta(1 + r_b)(t / T_{max})H \delta_h (1 - \epsilon)[(c_h + (1 - \epsilon)c_n) - c_h^o] + \delta_j(c_j + \epsilon c_n) - 2\eta(1 + r_b)(t / T_{max})H \delta_h (c_h + c_n - c_h^o) + \psi \mu \theta[1 - A^2] \epsilon / \bar{c}_j = 0 \quad (C4)
\]

By rearranging terms,

\[
\frac{\partial E(v)}{\partial c_n} = \psi \beta F[(\alpha / (1 - A))\epsilon + (1 - \epsilon)] + (1 - \psi)\beta F \alpha - \beta_{NF} p_n + \psi \mu \theta[1 - A^2] \epsilon / \bar{c}_j + 2\eta(1 + r_b)(t / T_{max})H \delta_h [(\psi(1 - \epsilon) + (1 - \psi)) c_n^o - 2\eta(1 + r_b)(t / T_{max})H \delta_h [\psi(1 - \epsilon)\psi c_j^* - 2\eta(1 + r_b)(t / T_{max})H \delta_h [\psi(1 - \epsilon) c_h^* - 2\eta(1 + r_b)(t / T_{max})H \delta_h [\psi(1 - \epsilon)^2 + (1 - \psi)] + \delta_j \psi \epsilon^2 c_n^* = 0 \quad (C5)
\]

From (C2) and (C3),

\[
c_j^* = \frac{\beta F[(\alpha / (1 - A)] + \mu \theta[1 - A^2] / \bar{c}_j - \beta_{NF} p_j - \psi \epsilon c_n^*}{2\eta(1 + r_b)(t / T_{max})H \delta_j} \quad (C6)
\]

\[
c_h^* = c_h^o + \frac{\beta F + \mu \theta[1 - A^2] / \bar{c}_j - \beta_{NF} p_h}{2\eta(1 + r_b)(t / T_{max})H \delta_h} - (1 - \epsilon \psi) c_n^* \quad (C7)
\]

The substitution of (C6) and (C7) into (C5) implies:
\[ \beta_{\text{r}}[(\alpha - 1)(1 - \psi \varepsilon)] + \beta_{\text{NF}}[\psi \varepsilon p_{j} + (1 - \psi \varepsilon)p_{h} - p_{n}] - (1 - \psi \varepsilon)\mu \theta[1 - \Lambda^{2}] / \bar{c}_{j} \]

\[ -2\eta(1 + r_{b})(t / T_{\text{max}})H(\delta_{h} + \delta_{j})\psi(1 - \psi)\varepsilon^{2}c_{n}^{*} = 0 \]  \hspace{1cm} (C8)

In turn,

\[ c_{n}^{*} = \frac{\beta_{\text{r}}(\alpha - 1)(1 - \psi \varepsilon) + \beta_{\text{NF}}[\psi \varepsilon p_{j} + (1 - \psi \varepsilon)p_{h} - p_{n}] - (1 - \psi \varepsilon)\mu \theta[1 - \Lambda^{2}] / \bar{c}_{j}}{2\eta(1 + r_{b})(t / T_{\text{max}})H(\delta_{h} + \delta_{j})\psi(1 - \psi)\varepsilon^{2}} \]  \hspace{1cm} (C9)
REFERENCES


