Discrete Population Models for a Single Species: Quiz 2002

Question 4. (15 marks)

It has been suggested that a means of controlling insect numbers is to introduce and maintain a number of sterile insects in the population. One such model for the resulting population dynamics is

\[ x_{n+1} = \frac{RMx_n^2}{(R-1)Mx_n^2 + Mx_n + S}, \]

where \( R > 1 \) and \( M > 0 \) are constant parameters, and \( S \) is the constant sterile insect population.

1. (a) Show that when \( S = 0 \) the difference equation can be simplified to

\[ x_{n+1} = \frac{Rx_n}{(R-1)x_n + 1}. \]

(1 mark)

(b) Show that when \( S = 0 \) the fixed points are given by \( x = 0 \) and \( x = 1 \).

(2 marks)
(c) Calculate the eigenvalues associated with the two fixed points and hence determine their stability. 3 marks
2. We now fix $R = 2$ and $M = 1$. The 3 figures in parts (b)–(d) show the graph

\[ x_{n+1} = f(x_n), \]
\[ = \frac{2x_n^2}{x_n^2 + x_n + S}, \]

for various values of $S$, and the straight line $x_{n+1} = x_n$.

(a) The figure on this page shows the graphs $y = f(x)$ and $y = x$ when $S = 0$.

i. Suppose that the initial population ($x_0$) is $x_0 = 0.5$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population. (1 mark)

ii. Explain what your cobweb plot shows. (1 mark)

iii. How would your answer to part (ii) change if you were to chose a different value for $x_0$ with $0 < x_0 < 1$? (1 mark)
(b) The figure on this page shows the graphs $y = f(x)$ and $y = x$ when $S = 0.15$.

i. Suppose that the initial population $(x_0)$ is $x_0 = 0.5$. By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population.

![Cobweb diagram]

ii. Explain what your cobweb plot shows.
iii. How would your answer to part (b)(ii) change if you were to choose a different value for $x_0$ with $0 < x_0 < 1$? (1 mark)
(c) The figure on this page shows the graphs \( y = f(x) \) and \( y = x \) when \( S = 0.35 \).

i. Suppose that the initial population \( x_0 \) is \( x_0 = 0.5 \). By drawing successive iterations on the cobweb diagram below determine the long-term evolution of the population. 

(1 mark)
(d) Comment on the biological implications of your answers to parts (a-c) of this question. (2 marks)