Impacts on resource allocations when the relative importance of allocation criteria is changed, with reference to Australian universities

Rodney Nillsen, University of Wollongong

ABSTRACT. In 1998, the West report on higher education in Australia considered possible changes to the monies allocated to universities on the basis of research. One method of doing this, and its implications, were discussed by David Phillips in an article in *The Australian*. The questions raised are treated here as an abstract resource allocation problem, and the discussion is not neccesarily restricted to universities. Using simple mathematics, the situation is analysed in somewhat more generality than in the article of Phillips, and the qualitative implications assessed. There is also a discussion of how, depending upon the circumstances, the parameters of the general procedure may be adjusted to avoid unacceptable outcomes, or to achieve ones considered desirable. The formulation of resource allocation problems in general mathematical terms potentially provides a greater understanding of the strengths and deficiencies of particular procedures and an appreciation of possible alternatives. In Australian public policy mathematics is generally seen as a secondary tool for data accumulation and associated arithmetical calculations. It is hoped that this paper may serve to show the potential for mathematical thinking to contribute to obtaining broad, qualitative insights into questions of public policy, while nevertheless maintaining the rigour and intellectual clarity associated with the discipline.

1. Introduction

A funding agency allocates monies to a number of recipients. There is a set of criteria by which different proportions of the total funds are allocated amongst the recipients. For each criterion, a definite sum of money is set aside to be allocated amongst the recipients on the basis of how each recipient meets that critierion. The total amount set aside for allocation according to a given criterion reflects the importance the funding agency places upon that criterion in relation to the other criteria. The recipients may have strengths regarding some criteria and have weaknesses regarding others, and these areas of strength and weakness may vary from one recipient to another.

One day, the funding agency decides to alter the relative importance it places upon the criteria—some criteria are to be increased in importance, and others are to be decreased. So, in relative terms, more funds are to be allocated on the basis of some of the criteria and less on the basis of some of the other criteria.

Also, the funding agency considers varying the total amount of funds it allocates to the recipients. The following questions then arise. (i) Under the new allocations, how are the recipients affected? (ii) How does the total funding for each recipient change? (iii) How does the proportional funding for each recipient change? (iv) Does the perceived status of a recipient change under the new allocation? (v) If the total effect of the changes is considered to have had unforeseen and undesirable consequences, is there a different or fairer method of allocation which is more suited to achieving desired outcomes?

We would expect, for example, that if a particular recipient is very strong in relation to a criterion to which more funds are to be allocated, then that recipient will receive more total funds if the other factors remain constant. But what is the *precise* manner in which the total funding to the recipient depends upon the change to placing more relative importance upon a particular criterion? If we *do* know the precise manner in which funding to the recipients depends upon the relative importance of the criteria and the total funding available, can we decide how much variation in outcomes can be achieved by varying the parameters of the process?

The above type of situation was encountered in 1998 when, in the West Report[1] on tertiary education in Australia, options were proposed for changing the proportion of funds given to universities on the basis of research criteria. An article by David Phillips, a former Head of the Higher Education Division of the Department of Employment, Education, Training and Youth Affairs, on the consequences of implementing these options, appeared in *The Australian* newspaper[2]. Phillips considered the implications of increasing the total amount allocated to universites for research (called the "research quantum"). Assuming that the funding for both teaching and research purposes to the totality of universities remained constant, this would have meant the total allocation to the universities for teaching would have had to decrease. Phillips calculated the effects upon the total allocations to 36 universities for every 1% increase in the research quantum, pointing out that the effects could be significant[3]. For example, for every 1% increase in the research quantum, he calculated that Melbourne University would gain \$3.2 million and the University of Western Sydney would lose \$1.6 million.

Using simple mathematics, this paper analyses the situation which arose in 1998, as discussed in the article by Phillips, but with some greater generality. Note that the analysis here applies to an abstract resource allocation problem, not simply to the specific one considered by Phillips. Its conclusions are valid in any situation satisfying the assumptions. So, it is only for convenience and immediacy that we consider the funding agency to be the Commonwealth Government, and refer to the recipients as "universities", and to the criteria as being "research" and "teaching".

In the analysis, we consider the allocation of funds to a number n of recipients subject to *two* criteria. This is a special case of the more general resource allocation problem described above. A more elaborate analysis could be carried out for the more general problem, but will not be attempted here. The changes in comparative allocations are described by what are known mathematically as "linear transformations" and ratios of such transformations. These give an exact form to what happens under any change in the parameters of this procedure of allocation.

This paper is intended to be accessible to people with some mathematical background but who are not necessarily mathematicians. Some familiarity with mathematical equations, summations and inequalities is assumed. Also, a capacity to follow an abstract argument involving algebraic symbols would be a considerable advantage. Otherwise, the reader may care simply to note the comments at the end of Section 2 and the non-mathematical comments in Section 3. in the paper, particular attention is paid to explaining in ordinary language the meaning of the mathematical argument.

2. Derivation of equations and inequalities

In a hypothetical country, a government has allocated an amount $\$T$ of funds to the whole university system. This and other amounts will be denoted without the \$ sign, as use of this sign is irrelevant to the analysis. The total funds T are split into two parts, one for teaching and the other for research. The total amount allocated for research is a, and the total amount allocated for teaching is b. So,

$$a + b \;=\; T. \tag{1}$$

Let us put

$$\rho \;=\; \frac{a}{b}. \tag{2}$$

Then ρ, being the ratio of the total research to the total teaching funds, represents the "overall balance" between research and teaching as far as the government is concerned. Alternatively, ρ represents the relative importance to government of the research and teaching functions of the universities. ρ can be thought of as a nominal "average index of research commitment" taken across all universities. We shall call ρ the *research index norm.*

Let us suppose that there are n universities, which we refer to as university 1, university 2, and so on. University j currently receives an amount a_j for research and an amount b_j for teaching. Thus,

$$\text{total current funds for university } j \;=\; a_j + b_j.$$

Also, the sum of the amounts for research at each university must equal the total amount for research, and similarly for teaching. Hence,

$$\sum_{j=1}^{n} a_j \;=\; a \quad \text{and} \quad \sum_{j=1}^{n} b_j \;=\; b. \tag{3}$$

The government has criteria by which it allocates a_j to university j for research, based upon what it considers to be the research performance of university j when compared with the other universities. The allocation of b_j to university j is based upon the number of students in university j. As far as the government is concerned, the criteria for allocating research funds reflect the extent to which the individual universties are, or perhaps should be, committed to research. (Note that the actual criteria or means of determining allocations for teaching and research to each institution are not relevant to the present analysis, but potentially could be the subject of a separate analysis.) The extent to which a university j can be considered a "research" university is measured for government purposes by the ratio a_j/b_j, for the higher the proportion of research funds a university has, the more it is considered to be a "research" university. We put

$$\rho_j = \frac{a_j}{b_j}, \qquad \text{for } \; j = 1, 2, \ldots, n, \tag{4}$$

and call it the *research index* for university j, relative to the funding described by (1) and (3). This formula is not the only one which could be taken to reflect the research commitment of a university; another one could be the ratio of the research funds to the total funds for the university $\big($that is the ratio $a_j/(a_j+b_j)\big)$, but in any case the method of measurement should depend only on the ratio a_j/b_j. If university j does no research and all teaching, then $a_j = 0$ and $\rho_j = 0$; while if university j does no teaching and all research, ρ_j is infinity. In practice, every university has some commitment to both research and teaching, and so it is convenient and legitimate for us to assume that $a_j > 0$ and $b_j > 0$ for all $j = 1, 2, \ldots, n$. Thus ρ_j is a number and it is strictly positive. In this country, status is attached to doing research, so the universities are very concerned if there are any changes in the system which reduce their value of ρ_j in relation to other universities, unless perhaps this is compensated for by a total increase in their funds.

A "trivial" case occurs if

$$\rho_1 \;=\; \rho_2 \;=\; \cdots \;=\; \rho_n,$$

for then it will happen that $\rho = \rho_j$ for all $j = 1, 2, \ldots, n$, all the universities have the same balance between teaching and research, and only differ in their relative size. In this case, under the following analysis, all universities would be affected in the same way. This case does not arise in practice. So, we will assume that not all of $\rho_1, \rho_2, \cdots, \rho_n$ are equal, although some of them may be. In this latter case, it follows from (1), (3) and (4) there are at least one j and one k such that

$$\rho_j > \rho \;\text{ and }\; \rho_k < \rho.$$ [4]

Apart from this, it should be noted that the assumptions restrict neither the number of values of j such that $\rho_j > \rho$ nor the number of values of k such that $\rho_k < \rho$. So, for example, there may be only one value of j such that $\rho_j > \rho$, or there may be up to $n-1$ values of j such that $\rho_j > \rho$ [5].

The government is considering allocating a different total amount for the whole system, and a different total amount for research. The universities would receive new research allocations which would be determined in proportion to the existing research allocations, and similarly for teaching. Essentially, this means that the research and teaching criteria would not change, but the amounts allocated under these criteria may change. Note, for example, that there are options such as increasing the total funding while decreasing funding for teaching (or research), and decreasing the total funding while increasing funding for teaching (or research).

We express the situation in mathematical terms as follows. Denote the new total amount for the system by T', the new total amount for research by a', the new total amount for teaching by b', the new research allocation at university j by a'_j, and the new allocation for teaching at university j by b'_j. In line with (1) and (3) above, we must have

$$a' + b' \;=\; T', \;\; \sum_{j=1}^{n} a'_j \;=\; a' \quad\text{and}\quad \sum_{j=1}^{n} b'_j \;=\; b'. \tag{5}$$

There is a number $\eta > 0$ such that

$$T' \;=\; \eta T. \tag{6}$$

If $\eta > 1$, there is to be an increase in the total funds for the system, but if $\eta < 1$, there is to be a decrease in the total funds available. The case $\eta = 1$ is when the funds remain the same, and this was the situation discussed in the article by Phillips.

As the new research allocations are to be in proportion to the present allocations, there is a number $\theta > 0$ such that

$$a' \;=\; \theta a \;\text{ and }\; a'_j \;=\; \theta a_j \;\text{ for } j = 1, 2, \ldots, n. \tag{7}$$

Also, as the new teaching allocations are to be in proportion to the present ones, there is a number $\phi > 0$ such that

$$b' \;=\; \phi b \;\text{ and }\; b'_j \;=\; \phi b_j \;\text{ for } j = 1, 2, \ldots, n. \tag{8}$$

There is a relationship between θ and ϕ owing to the fact that the sum of the new teaching and research allocations must equal the total funds available. Thus,

$$\theta a + \phi b \;=\; a' + b' \;=\; \eta T \;=\; \eta a + \eta b. \tag{9}$$

This gives, using (2),

$$\begin{aligned} \phi &= \frac{\eta a + \eta b - \theta a}{b} \\ &= \eta \frac{a}{b} + \eta - \theta \frac{a}{b} \\ &= \eta\rho + \eta - \theta\rho. \end{aligned} \tag{10}$$

It follows from (7), (8), (9) and (10) that

$$b' \;=\; \eta T - \theta a \;\text{ and that }\; b'_j \;=\; \Big(\eta\rho + \eta - \theta\rho\Big) b_j, \;\text{ for }\; j = 1, 2, \ldots, n. \tag{11}$$

What about the total allocation for university j? Assuming as we do that $b_j > 0$, the absolute change in allocation to university j is

$$\begin{aligned} a'_j + b'_j - a_j - b_j &= \theta a_j + \phi b_j - a_j - b_j \\ &= \theta a_j + \Big(\eta\rho + \eta - \theta\rho\Big) b_j - a_j - b_j, \;\text{ by }\; (11) \\ &= (\theta - 1) a_j + \Big(\eta\rho + \eta - \theta\rho - 1\Big) b_j \\ &= b_j \left((\theta - 1) \frac{a_j}{b_j} + \eta\rho + \eta - \theta\rho - 1 \right) \\ &= b_j \left((\theta - 1)\rho_j - (\theta - 1)\rho + \eta\rho + \eta - \rho - 1 \right) \\ &= b_j \left((\theta - 1)\rho_j - (\theta - 1)\rho + (\eta - 1)\rho + \eta - 1 \right) \\ &= b_j \left((\theta - 1)(\rho_j - \rho) + (\eta - 1)(\rho + 1) \right). \end{aligned} \tag{12}$$

Incidentally, in view of (3), (5) and (6) we must have

$$\sum_{j=1}^{n} a'_j + b'_j - a_j - b_j \;=\; (\eta - 1) T,$$

and it is an exercise to check this independently from (12), in which case the correctness of the calculations is confirmed.

Now, equation (12) shows that the increase or decrease in the funds for university j is proportional to b_j, the amount originally allocated to teaching at university j. However, (12) shows that the increase or decrease in funds to university j is not generally proportional to $\theta - 1$, which is the proportion by which the total funds allocated to research are increased. However, in the case when $\eta = 1$, (12) shows that the increase or decrease in funds to university j *is* proportional to $\theta - 1$, as noted by Phillips in the particular case he considered. Furthermore, when $\eta = 1$, (12) shows that the increase or decrease in funds to university j is also proportional to $\rho_j - \rho$.

Now, if $\theta \neq 1$, we have from (12) that

$$a'_j + b'_j - a_j - b_j = b_j(\theta - 1)\left(\rho_j - \rho + (\rho + 1) \cdot \left(\frac{\eta - 1}{\theta - 1}\right)\right). \tag{13}$$

Hence, in the case when $\theta > 1$, the total allocation for university j will increase if and only if

$$\rho_j \ > \ \rho - (\rho + 1) \cdot \left(\frac{\eta - 1}{\theta - 1}\right). \tag{14}$$

Alternatively, when $\theta > 1$, the occurrence of an increase in the total allocation for university j is equivalent to having

$$\rho_j \ > \ \left(\frac{\theta - \eta}{\theta - 1}\right)\left(\rho + \frac{1 - \eta}{\theta - \eta}\right). \tag{15}$$

The case $\eta = 1$ is the one considered by Phillips and is when the total funds allocated to the universities are unchanged. In this case we have from (13), (14) or (15) that the total allocation for university j will increase precisely when

$$\rho_j > \rho. \tag{16}$$

This rather neat result is not surprising, but even so I don't think it is immediately obvious from the allocation method itself. However, Phillips may have considered this result to be self evident when he expressed it in the following equivalent non-mathematical way: "Therefore, a university will be a winner from an increase in the size of the research quantum if its share of the research quantum is greater than its share of the teaching component of operating grants".

When we allow the total of available funds to change as well, inequalities (14) and (15) express the more complex situation as to when a university will receive a total increase. As each $\rho_j > 0$, it follows from (15) that when $\theta > 1$ *every* university will receive an increase in total funds precisely when

$$\big(\theta - \eta\big)\rho \ \leq \ \eta - 1.$$

This is impossible if $\eta \leq 1$ and $\theta > \eta$, but will hold when $\eta \geq \theta$. More generally, when $\theta > 1$, we see that every university will receive a total increase if

$$\eta \ \geq \ \frac{1 + \rho\theta}{1 + \rho}.$$

So, if the government wishes no university to have a decrease in total funds for political reasons, this inequality tells us the proportion by which the total funds need to be increased in order to outweigh the decrease in research funds to those universities with a lower research index.

What about the *proportional* increase or decrease of funds to each university? For university j this corresponds to the number

$$p_j \quad = \quad \frac{a'_j + b'_j - a_j - b_j}{a_j + b_j}.$$

Now,

$$\begin{aligned}
p_j &= \frac{a'_j + b'_j - a_j - b_j}{a_j + b_j} \\
&= \frac{(\theta - 1)a_j + \Big(\eta\rho + \eta - \theta\rho - 1\Big)b_j}{a_j + b_j} \\
&= \frac{(\theta - 1)\rho_j + \Big(\eta\rho + \eta - \theta\rho - 1\Big)}{\rho_j + 1} \\
&= \frac{(\theta - 1)(\rho_j + 1) - (\theta - 1) + \Big(\eta\rho + \eta - \theta\rho - 1\Big)}{\rho_j + 1} \\
&= \theta - 1 + \frac{\eta\rho + \eta - \theta\rho - \theta}{\rho_j + 1} \\
&= \theta - 1 + (\eta - \theta)\left(\frac{\rho + 1}{\rho_j + 1}\right) \\
&= (\theta - 1)\left(1 - \frac{\theta - \eta}{\theta - 1} \cdot \frac{\rho + 1}{\rho_j + 1}\right). \qquad (17)
\end{aligned}$$

We see from (17) that if $\theta > \eta$ and $\theta > 1$, then the larger the value of ρ_j, the larger the proportional increase in the funds for university j. However, if $\theta < \eta$ and $\theta > 1$, then the larger the value of ρ_j, the smaller the proportional increase in the funds for university j. Note also that from (17) it follows that the relative increase in p_j depends upon neither a_j nor b_j directly, but only upon their ratio ρ_j.

Now make the definition that

$$f(x) = (\theta - 1)\left(1 - \frac{\theta - \eta}{\theta - 1} \cdot \frac{\rho + 1}{x + 1}\right). \qquad (18)$$

The point is that $p_j \; = \; f(\rho_j)$, so that the properties of function f tell us how the p_j behave. Using (18), we see that the derivative of f is the function f' given by

$$f'(x) = \frac{(\theta - \eta) \cdot (\rho + 1)}{(x + 1)^2}. \qquad (19)$$

The derivative tells us how sensitive $f(x)$ is to changes in x. We see from (19) that if $\theta > \eta$, $f'(x) > 0$ for all x, which implies that $f(x)$ increases as x increases. That is, if the proportional change in the total research allocation is greater than the proportional change in the total allocation, the universities with the higher research indices benefit more in proportional terms. However, we can say more than this. For, (19) shows that when $\theta > \eta$, $f'(x)$ is larger for small values of x. That is, $f(x)$ changes more rapidly for small values of x. So in this case, for universities which have a low value of ρ_j, slight differences between the values of the ρ_j have greater effects on the proportional changes under the new allocations—small differences in the research indices will have larger proportional effects on these universities in comparision with the others. However, it needs to be emphasised that the extent of these effects depends strongly on the actual values of the parameters in the problem.[6] Note also that although the emphasis here has been, rather optimistically, on when the allocated funds may increase, a formally similar analysis applies when the funds decrease. For, from the mathematical viewpoint, a decrease in funds is simply a negative increase in funds.

3. Obtaining desirable or acceptable outcomes

In many questions of public policy, it is considered important to have a seemingly rational or objective basis on which policy is putatively based, and such a basis is the one usually presented publicly as the reason for the policy. Such is public respect for reason, even today when superficial perception dominates substance. When it comes to the use of mathematics, its formulas give an appearance of objectivity which, in some respects, is fully justified. For, conclusions deriving from the mathematics are objective in that they derive necessarily from the mathematical formulation of the problem, regardless of the prejudices or preferences of the person following the analysis. In this sense, the use of mathematics removes the individual or an interest group from the analysis. It is this independence of the analysis from individual or group pressure which gives mathematical argument its force and objectivity. The fact that relatively few people understand mathematical arguments can also make it difficult for people to argue against this apparent objectivity, even when they dislike the conclusions and their implications, and this is often used by politicians and others to bolster arguments in favour of certain social policies.[7].

It needs to be noted that in any mathematical analysis of actual policies the underlying assumptions of the analysis may appear to be reasonable and fair, but the analysis may reveal implications which were not apparent in the original assumptions. These implications may suggest that a wider range of criteria should be brought to bear in making any judgment of the fairness and effectiveness of the procedure. At one extreme, these additional criteria may be purely cynical and political, but on the other hand they may help to genuinely improve the fairness of the procedure. (We should always bear in mind that no system of human investigation, not even mathematics, is completely removed from the motives of the investigators.) However, the point to note here is that the procedure is not grasped intellectually, let alone ethically, merely by understanding the immediate techniques of how to carry it out. We grasp the procedure better when we can comprehend its effects as a whole, and understand how outcomes vary with variation in the parameters. Then, we may grasp the procedure better still when we can assess its range of possible outcomes against other alternative procedures. Potentially, in the right circumstances, this is one thing which an abstract mathematical analysis of a problem can provide.[8] This approach requires a conscious use of mathematics as more than a mere tool of calculation, but rather as a precise means of critical reflection and of exploring possibilities. It also requires us to "distance" ourselves from the procedure, and to consider it dispassionately as to its fairness and appropriateness. When pursued at a sufficently high or intense level, involving the whole person, this type of analysis shatters the mental barrier, common in Australia, which limits education to the acquisition of information. Once the whole person becomes involved, the analysis can take on an ethical and moral dimension whose justification lies beyond its immediate political or corporate aims, an effect accentuated in mathematics because of the objectivity adhering to it, in the sense alluded to above. Of course, this dimension will vanish the instant the analysis is regarded, even implicitly, as no more than an instrument for achieving short-term political gain or marketing advantage[9].

In the present context, whereas Section 2 dealt largely with the direct consequences of the assumptions, in the present section we shall comment on what can be termed the “inverse problem”. That is, to what extent can the procedures and parameters be changed or “manipulated”, to avoid unacceptable outcomes? Conceptually, we can think of this as interchanging the rôles of the input and output parameters. If we can look at the set of all possible policy outcomes, for the moment in mere political terms, it may be possible to a produce a policy acceptable to a government, but one which simultaneously would gain more general acceptance than otherwise. Apart from this, a fuller and more abstract comprehension of how the parameters in the process affect the policy outcomes can contribute to understanding how inequities or advantage may occur, and lead to a critique which may produce a better procedure.

Our aims here are more strictly limited, but may suffice to illustrate the possibilities. In the analysis in Section 2, the outcomes in a specific situation will depend very much upon the actual distribution values of $\rho_1, \ldots, \rho_n$. Assuming that the total funds are to remain constant, (16) shows that only those universities which have a research index which is greater than the research index norm will receive any funding increase. If the number of such universities is few, this may be a politically unacceptable outcome. However, if there are several universities whose research index is less than the research index norm, it may be possible to increase the total funding by a relatively small amount, while nevertheless having the effect of giving more universities a nett increase in funds. This could make it easier to argue that the new allocations would be fair. To illustrate this, observe that the inequality (14) can be written as

$$\eta \ > \ 1 + \left(\frac{\theta - 1}{\rho + 1}\right)(\rho - \rho_j). \tag{20}$$

Thus, if $\theta > 1$, inequality (20) precisely expresses the situation when university j will receive an increase in its total allocation under the new system. Now in (20), if $\rho_j < \rho$ but ρ_j is sufficently close to ρ, the right hand side of (20) will be a number larger than 1 but close to it. Thus, in such a case, η may be chosen to be only a little larger than 1, but nevertheless (20) would hold. Thus, by making only a small increase in the funds to the whole system, university j would now receive an increase. This analysis may apply simultaneously to several universities, depending on whether there are many values of ρ_j which are less than ρ but still sufficently close to it. Inequality (20) enables us to estimate precisely how we would need to adjust the input parameter θ so as to give specified universities an overall funding increase in the changed system.

Now, let us consider how the research indices would be affected under the new system. The research index for university j under the new allocation is

$$\begin{aligned} \rho'_j &= \frac{a'_j}{b'_j} \\ &= \frac{\theta a_j}{\phi b_j}, \quad \text{by (7) and (8),} \end{aligned}$$

$$
\begin{aligned}
&= \frac{\theta a_j}{\Big(\eta\rho + \eta - \theta\rho\Big) b_j}, \quad \text{by (10),} \\
&= \frac{\theta \rho_j}{\eta\rho + \eta - \theta\rho}.
\end{aligned}
$$

Hence,

$$
\frac{\rho_j'}{\rho_j} = \frac{\theta}{\eta\rho + \eta - \theta\rho}. \tag{21}
$$

This shows that under the new allocations, the ratio of the respective research indices is the same for *all* universities. It also follows from (21) that if

$$
\frac{\rho_j'}{\rho_j} > 1
$$

for university j, then the corresponding ratio of new and old new research indices is greater than 1 for *all* universities. In fact, because we can deduce from (21) that

$$
\frac{\rho_j'}{\rho_j} - 1 \ = \ \frac{(\theta - \eta)(\rho + 1)}{\eta\rho + \eta - \theta\rho}, \tag{22}
$$

we see that the new research indices are greater than the old precisely when $\theta > \eta$. Thus, (22) shows that when $\theta > \eta$, every university becomes more of a research university than before, as measured by the inequality $\rho_j' > \rho_j$, even though some universities will receive less funding than before. So, when $\theta > \eta$, the effect of the procedure is not simply to "reward" the stronger research universites but also to tilt the research balance in *every* university more towards research. This may not necessarily have been an aim of the original procedure.

Any judgment on the appropriateness of an allocation procedure depends upon what it is intended to achieve. If the main aim is to encourage each university in the whole system to place more emphasis on research, in part by rewarding those institutions which are currently strongest in research, then the procedure discussed above may be considered reasonable. On the other hand, if the aim is to *differentiate more* between universities on the basis of whether they are "research" or "teaching" universities, and to encourage the former to develop their research and the latter to develop their teaching, then the procedure discussed here seems to be inadequate. A further mathematical analysis, on more complicated lines, would be needed to develop more appropriate procedures for these circumstances. Finally, note that these two possible aims of an allocation procedure by no means exhaust the range of possibilities.

Notes

1. Learning for Life, A Review of Higher Education Financing and Policy, *Department of Employment, Education, Training and Youth Affairs, Commonwealth of Australia, 1998.* Recommendation 37 of the report was: "That the Government review the size and basis for allocating the research quantum...". Page 164 of the report considers 4%, 6% and 10% as possible proportions of the total budget for universities which could be based upon research.

2. David Phillips, West's Quantum Mechanics, *The Australian*, July 1, 1998, page 41. The article is attached as an appendix.

3. The total allocations are only proportional to each percentage increase in the research quantum when the totality of all funds to be allocated is unchanged. In general, a 2% increase in the research quantum does not have twice the effect of a 1% increase. See the comments following equation (12).

4. For if $\rho_j > \rho$ for all $j = 1, 2, \ldots, n$, we have

$$\begin{aligned} \rho &= \frac{a}{b}, \quad \text{by (2)}, \\ &= \frac{\sum_{j=1}^{n} a_j}{\sum_{j=1}^{n} b_j} \\ &= \frac{\sum_{j=1}^{n} \rho_j b_j}{\sum_{j=1}^{n} b_j} \\ &> \frac{\sum_{j=1}^{n} \rho b_j}{\sum_{j=1}^{n} b_j}, \quad \text{by (4)}, \\ &= \rho, \end{aligned}$$

which is not possible. So, $\rho_j < \rho$ for some j.

5. In fact, let $\rho > 0$ and let $\rho_1, \rho_2, \ldots, \rho_n > 0$ be such that there are j, k with $\rho_j > \rho > \rho_k$. Then there are $a_1, \ldots, a_n, b_1, \ldots, b_n > 0$ such that

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = \rho \quad \text{and} \quad \frac{a_i}{b_i} = \rho_i, \quad \text{for all} \quad i = 1, 2, \ldots, n.$$

Here is a proof, for those with the mathematical background. Let

$$A = \Big\{ (b_1, b_2, \ldots, b_n) : b_j > 0 \text{ for all } j = 1, 2, \ldots, n \Big\}.$$

Let $f : A \longrightarrow [0, \infty)$ be the function given by

$$f\Big(b_1, \ldots, b_n\Big) = \frac{\rho_1 b_1 + \rho_2 b_2 + \cdots + \rho_n b_n}{b_1 + b_2 + \cdots + b_n}.$$

Because of the condition that $\rho_j > \rho > \rho_k$, there are $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ in A such that

$$f\Big(y_1, \ldots, y_n\Big) < \rho < f\Big(x_1, \ldots, x_n\Big).$$

Put, for $0 \le t \le 1$,

$$\psi(t) = f\Big(tx_1 + (1-t)y_1, \ldots, tx_n + (1-t)y_n\Big).$$

Then ψ is continuous, and

$$\psi(0) = f\Big(y_1, \ldots, y_n\Big) < \rho < f\Big(x_1, \ldots, x_n\Big) = \psi(1).$$

By Bolzano's Theorem, there is a $s \in [0, 1]$ such that $\psi(s) = \rho$. Put $b_j = sx_j + (1-s)y_j$ for $j = 1, 2, \ldots, n$.

Then

$$\frac{\rho_1 b_1 + \rho_2 b_2 + \cdots + \rho_n b_n}{b_1 + b_2 + \cdots + b_n} = f\Big(b_1, \ldots, b_n\Big) = \psi(s) = \rho.$$

So if we put $a_j = \rho_j b_j$ for $j = 1, 2, \ldots, n$ we will have

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = \rho \quad \text{and} \quad \frac{a_i}{b_i} = \rho_i, \quad \text{for all} \quad i = 1, 2, \ldots, n,$$

as stated.

6. The Mean Value Theorem of the differential calculus is relevant here. What is at issue is to estimate $|f(x)-f(y)|$. The Mean Value Theorem asserts the existence of a point ξ between x and y such that

$$|f(x) - f(y)| \ = |x - y| \cdot \ \frac{(\theta - \eta) \cdot (\rho + 1)}{(\xi + 1)^2}.$$

7. Thus, for example, in relation to Commonwealth Government funding for private schools Dr David Kemp, the minister, has recently argued "...the SES [Socio-Economic Status] formula ...provides an objective measure based on the needs of communities". *The Australian*, 11/10/00, page 13. In fact, the objectivity in this case is highly questionable, not necessarily because of the formula itself, but because the conclusions the minister derives from the formula apparently are based upon the obviously false notion that a small and non-random sample taken from a large group is necessarily representative of the group as a whole.

8. Also, mathematics can identify errors which may occur in a shallow analysis of problems. The point has been well made by David moore and George Cobb (*American mathematical Monthly*, 107(2000), p.623): "Elaborate studies using complex statistics, when done without much background in mathematical or statistical thinking, are prone to errors that lie deeply buried in the awful details. This is of course true whenever complex mathematical models are automated and then used without adequate grounding in mathematics and in the substance that the mathematics describes".

9. Such issues are rarely comprehended, if at all, in public policy discourse in Australia, or even within the universities themselves. For example, Mr John Dawkins, a minister in a former government, reportedly expressed the view that he could not understand why anyone would study at university except to get a job. Although it is rarely spelled out in these crude terms, I think it is fair to say that this has been a "subconscious" or implicit view of successive governments over the last 12 years. Despite the prevailing rhetoric concerning "problem solving skills", the effect is that students come to think of education as the mere acquisition of information, as presumably befits the "information age". In many respects, such views of education and their effects coincide remarkably with those described by the Controller in Aldous Huxley's novel "Brave New World", published about 80 years ago.

Rodney Nillsen

School of Mathematics and Applied Statistics

University of Wollongong

Northfields Avenue

Wollongong, NSW 2522

AUSTRALIA

email: rodney_nillsen@uow.edu.au