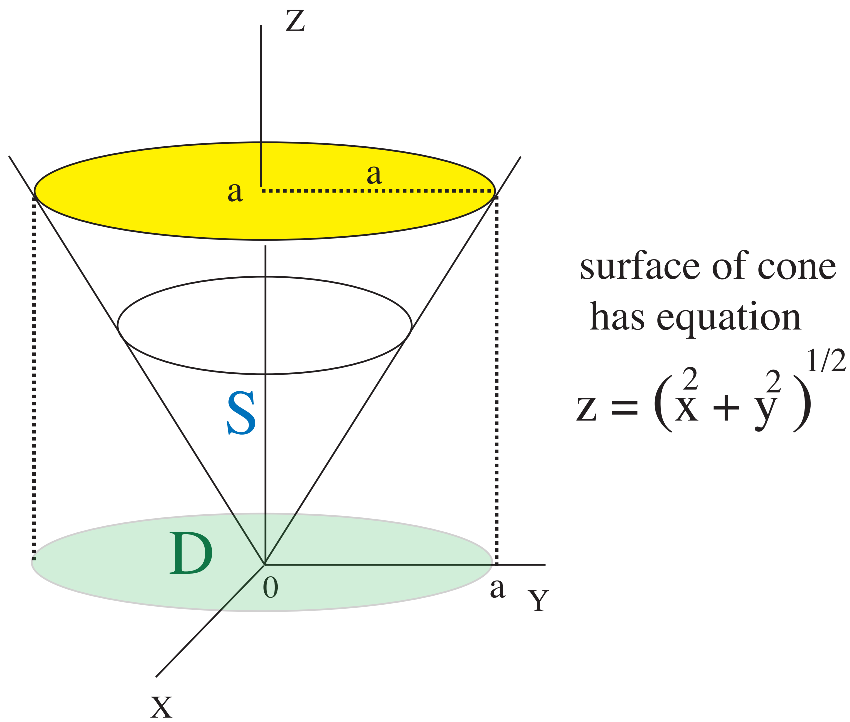
MATH 201:AN EXAMPLE ON SURFACE INTEGRALS



The cone in the picture has equation

$$h(x,y)=\sqrt{x^2+y^2}.$$

The surface S consists of the surface of the outside of the cone from $z=0$ to $z=a$, where $a>0$ is given – note that S does **not** the include the circular flat "top" of this part of the cone.

Now, let $\phi : S \longrightarrow \mathbb{R}$ be a given function. By results in the lecture notes,

$$\int\int_S \phi\ dS = \int\int_D \phi(x,y,h(x,y))\sqrt{1+\left(\frac{\partial h}{\partial x}\right)^2+\left(\frac{\partial h}{\partial y}\right)^2}\ dxdy.$$

Here D is the region in the XY plane that lies *underneath* the surface over which we are integrating. We see that D is a circle with its interior, the circle has centre 0 and radius a, so that

$$D=\{(x,y) : x^2+y^2\le a\}.$$

We see that

$$\begin{aligned}\int\int_S \phi\ dS &= \int\int_{x^2+y^2\le a^2} \phi(x,y,h(x,y))\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\ dxdy \\ &= \sqrt{2}\int\int_{x^2+y^2\le a^2} \phi(x,y,h(x,y))\ dxdy\end{aligned}$$

ROD NILLSEN, May 2008