

Math 201 EXAMPLE ON CALCULATING THE DERIVATIVE

Recall that for $x \in \mathbb{R}^n$ we write

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

LET $f : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the function given by

$$f(x) = f \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1^2 - x_3^2 \\ \sin(x_1x_2x_3x_4) \\ x_1 + x_2 + x_3 + x_4 \end{pmatrix}.$$

The coordinate functions f_1, f_2, f_3 are given by

$$f_1(x) = x_1^2 - x_3^2, \quad f_2(x) = \sin(x_1x_2x_3x_4) \quad \text{and} \quad f_3(x) = x_1 + x_2 + x_3 + x_4.$$

So we have

$$D_1f_1(x) = 2x_1, \quad D_2f_1(x) = 0, \quad D_3f_1(x) = -2x_3, \quad D_4f_1(x) = 0,$$

$$D_1f_2(x) = x_2x_3x_4 \cos(x_1x_2x_3x_4), \quad D_2f_2(x) = x_1x_3x_4 \cos(x_1x_2x_3x_4),$$

$$D_3f_2(x) = x_1x_2x_4 \cos(x_1x_2x_3x_4) \quad \text{and} \quad D_4f_2(x) = x_1x_2x_3 \cos(x_1x_2x_3x_4),$$

$$D_1f_3(x) = D_2f_3(x) = D_3f_3(x) = D_4f_3(x) = 1.$$

Thus,

$$f'(x) = \begin{pmatrix} 2x_1 & 0 & -2x_3 & 0 \\ x_2x_3x_4 \cos(x_1x_2x_3x_4) & x_1x_3x_4 \cos(x_1x_2x_3x_4) & x_1x_2x_4 \cos(x_1x_2x_3x_4) & x_1x_2x_3 \cos(x_1x_2x_3x_4) \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

In particular,

$$f' \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad f' \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

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