

## MATH 201: things to think about in week1

The first two questions involve revision ideas from first year, covered in Sections 2.1 to 2.5 of the notes. The third question gives a hint of some of the new ideas in Math201.

1. Let  $f$  be the function given by

$$f(x) = x^3 - x, \text{ for } -\infty < x < \infty.$$

(i) Sketch the graph of  $f$ , and calculate the maximum and minimum values of  $f$ .

(ii) Show that the equation

$$f(x) = 1$$

has a unique solution (that is, exactly one solution) for  $x$ .

(iii) Show that the equation

$$f(x) = -1$$

has a unique solution for  $x$ .

(iv) Show that if

$$\frac{-2}{3\sqrt{3}} < y < \frac{2}{3\sqrt{3}},$$

then the equation

$$f(x) = y$$

has three (distinct) solutions for  $x$ , while if

$$|y| \geq \frac{2}{3\sqrt{3}},$$

the equation has a unique solution for  $x$ . Illustrate what is going on in your graph.

(v) What is the range of the function  $f$ ? Is the function one-to-one?

(vi) Does the function  $f$  have an inverse? Can you change the domain of  $f$  so that the new function obtained has an inverse? What would you change the domain to?

2. Now, we have

$$\frac{dx^2}{dx} = 2x, \tag{1}$$

for all values of  $x$ . If in this equation (1) we put  $x = 1$  we get

$$\frac{d1}{d1} = 2. \tag{2}$$

Now, as

$$\frac{dx}{dx} = 1,$$

for all values of  $x$ , putting  $x = 1$  gives

$$\frac{d1}{d1} = 1. \tag{3}$$

Comparing (2) and (3) we get

$$1 = 2.$$

Explain any errors in the above reasoning. What does this tell you, if anything, about the notation

$$\frac{dy}{dx}$$

for the derivative of a function  $y$ ? Does using the notation  $y'(x)$  avoid problems such as the above?

**3.** Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be given by

$$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}.$$

Now, let  $L$  denote the line in  $\mathbb{R}^2$  whose equation is given by

$$y = x.$$

Draw the graph of  $L$ , and explain why  $f$  maps  $L$  onto the  $Y$ -axis.

What line does  $f$  map onto the  $X$ -axis?

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