MATH 201 2008

ANSWERS TO EXERCISES 6.7

1. At a general point on the curve, the tangent vector is $(-a\sin t, b\cos t)$. At $(a/\sqrt{2}, b/\sqrt{2}$ it is $(-a/\sqrt{2}, b/\sqrt{2})$.

2. $D_1 f(x,y,z) = 2x$, $D_2 f(x,y,z) = -3$, $D_3 f(x,y,z) = 4z^3$, $f'(x,y,z) = (2x, -3, 4z^3)$.

$$D_u f(x,y,z) = \left(\frac{2x}{3}, -2, -\frac{8}{3}z^3\right), D_u f(1,1,-2) = \left(\frac{2}{3}, -2, \frac{64}{3}\right).$$

3. There is a mistake in the question, because the formula given for the function f is not consistent with the statement that $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$. The directional derivative is only worked out for functions taking values in $\mathbb{R}$, so maybe just ignore this question.

5. $\phi(x,y) = -y^2x + x^2y$, $\mathrm{div}F(x,y) = 2y - 2x$.

7. $\mathrm{div}F(x,y,z) = y^2 - z^2 + 2xz$. $\mathrm{curl}F(x,y,z) = (2yz, -2xz, -2xy)$.

ANSWERS TO EXERCISES 7.11

1. -36.9
2. -2π
4. 7
6. 24
7. 150
8. 36π
9. $\frac{2\pi}{3}$
10. 18π
11. 320π
12. $\frac{32}{3}$
13. (a) 3, (b) $4\pi a^3$
14. $\frac{a^5}{30}$
15. 128
16. π
17. 324/5
18. 4/3
19. $(128, -24, 384)$
20. $4\pi\sqrt{\frac{2}{3}}$
21. $\pi ab/8$
22. (a) 0, (b) This is longer than intended. The problem is that S has 6 faces, and the integral over each face needs to be worked out separately–it is not clear that the Divergence Theorem can be easily used. In the XY-plane the face of S is the rectangle $S_1 = [0,1]\times[0,2]$, and the surface above it is that part of the plane given by $2x+y+2z = 6$. Call this upper part of the surface S_2. Then,

$$\int\int_{S_1} (4x + 3y)dxdy = 10,$$

and

$$\int\int_{S_2}(4x+3y-2z)dS = \frac{\sqrt{5}}{2}\int_0^2\left(\int_0^1\left(4x+3y-2\left(\frac{6-2x-y}{2}\right)\right)dx\right)dy,$$

etc etc.

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