**EXAMPLE.**

LET $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be given by

$$f(x, y) = (x + y, 2x - 3y).$$

We have

$$f'(x, y) = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}.$$

To calculate the inverse of $f$ we have to solve the equations

$$u = x + y, v = 2x - 3y. \qquad (*)$$

to get $x, y$ in terms of $u, v$. We find, and this can easily be checked that

$$x = \frac{1}{5}(3u + v) \text{ and } y = \frac{1}{5}(2u - v).$$

Thus, $f$ is one-to-one, $f$ has an inverse that is given by

$$f^{-1}(u, v) = \left(\frac{1}{5}(3u + v), \frac{1}{5}(2u - v)\right).$$

Thus,

$$(f^{-1})'(u, v) = \frac{1}{5}\begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}^{-1} = (f'(x, y))^{-1}.$$

Thus, if $u, v$ are given by $(*)$, we have

$$(f^{-1})'(u, v) = (f'(x, y))^{-1},$$

as predicted by the Inverse Function Theorem.

Rod Nillsen
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