Answers to exercises in Chapter 3, Exercises 3.4 of the MATH201 lecture notes, 2008

1. Direction of the line is given by $(2,1,-1)$. A vector orthogonal to the line is $(1,-2,0)$.

2. Points on the line have the form $(1,-4,-1)+t(-1,1,2)$, for $t \in \mathbb{R}$. That is, points on the line are those of the form $(1-t,-4+t,-1+2t)$, for $t \in \mathbb{R}$.

Points on the line parallel to this line and going through $(3,4,0)$ are $(3,4,0)+t(-1,1,2)$, for $t \in \mathbb{R}$. That is, they are of the form $(3-t,4+t,2t)$.

4. If one of x, y is a multiple of the other, then $sx+ty$ is on the line going through x and y. Otherwise, a condition on s, t for w to lie on the given line is $s+t=1$. [It helps to draw a picture, using the paralleogram law for addition of vectors, that illustrates that adding sx and ty does not in general give a point on the line going through x and y.]

5. A vector x is a **unit vector** if $|x|=1$. If $x \neq 0$, $x/|x|$ is a unit vector.

(i) $(-1,1,2)/\sqrt{6}$, (ii) $(1,5,-2)\sqrt{30}$, (iii) $(1,-2,0)/\sqrt{5}$.

6. (i) $a=(1,1,1)$, $b=(1,0,0)$, $c=(\alpha,\beta,\gamma)$.

(ii) $\gamma \neq 2$. [Note the question means find γ such that a, b, c lie in a common plane through the origin.]

7. Plane has equation $y-z+1=0$. $(3,2,3)$ satisfies this equation. $(0,1,-1)$ is normal to the plane. Points on the given line are $(3,2+t,3-t)$, for $t \in \mathbb{R}$.

8. Two points on the line are $(0,7/2,5/2)$ and $(5/3,-2/3,0)$. Points on the line are those of the from $(0,7/2,5/2)+t(5/3,-25/6,-5/2)$, $t \in \mathbb{R}$. [Maybe simplify this.]

9. Points on $C \cap P$ are those points of the form $(x, 1-x, \sqrt{5x^2-8x+4})$ for $x \in \mathbb{R}$.

10. Note that if x, y, z lie in a common plane P through the origin, then $0, x, y, z \in P$. So y, z are in the direction of the plane.

11. See figures 3.6, 3.7 & 3.8 for some of this.

12. (i) elliptic paraboloid, (ii) ellipsoid, (iii) hyperboloid, (iv) cicular cylinder in the y direction, (v) parabolic cylinder in the x direction, (vi) parabolic cylinder in the y direction, (vii) plane, (viii) surface of sphere of radius a and centre $(0,0,a)$, (ix) a cone centred at $(0,0,0)$, (x)hyperboloid (see Figure 3.6), (xi) parabolic cylinder in the x direction, (xii) a plane, (xiii) circular cylinder in the x direction, (xiv) a plane.