



## Random Intercepts Model

$$\text{weight}_{ij} = U_i + \beta_0 + \beta_1 \text{weeks}_{ij} + \varepsilon_{ij}$$

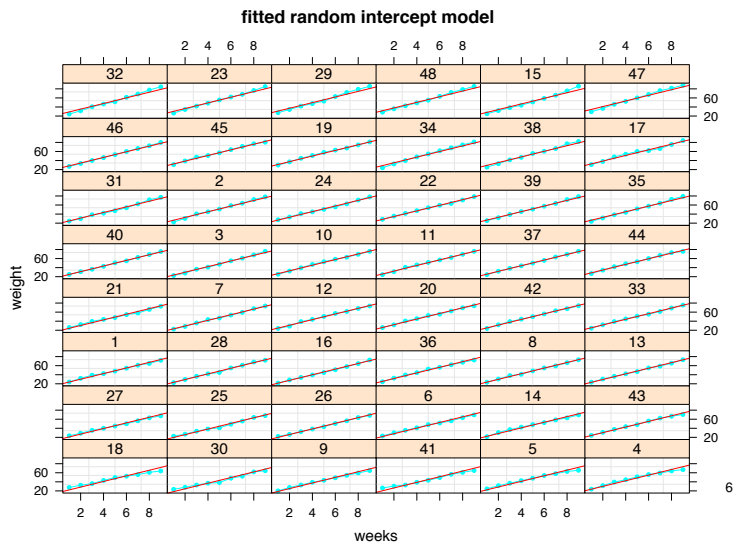
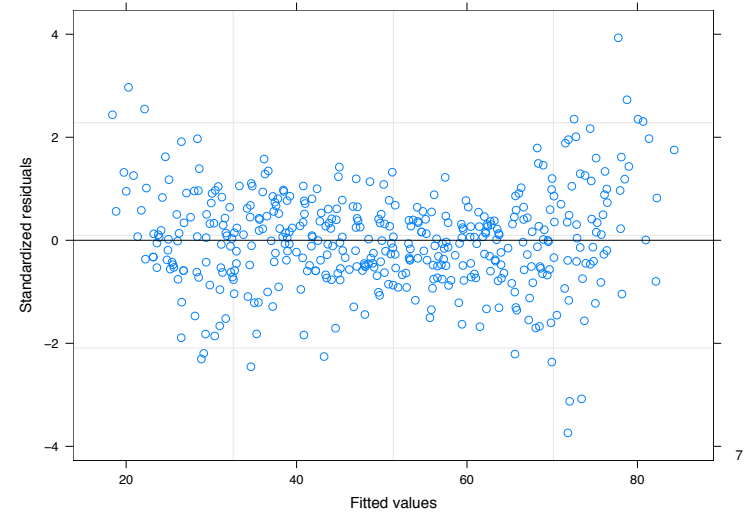
where

$$U_i \stackrel{\text{ind.}}{\sim} N(0, \sigma_U^2)$$

are independent of the

$$\varepsilon_{ij} \stackrel{\text{ind.}}{\sim} N(0, \sigma_\varepsilon^2).$$

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## Comments on Previous Two Slides

Close inspection of the fitted **red lines** shows that the **parallel line assumption**

inherent in the random intercept model is **too restrictive.**

This is confirmed in the **residual plot** which shows a pronounced

**bow tie pattern.**

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## Random Intercepts and Slopes Model

A remedy is to allow **each pig** to have  
**his/her own slope...**

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## Random Intercepts and Slopes Model II

$$\text{weight}_{ij} = U_i + \beta_0 + (\beta_1 + V_i) \text{weeks}_{ij} + \varepsilon_{ij}$$

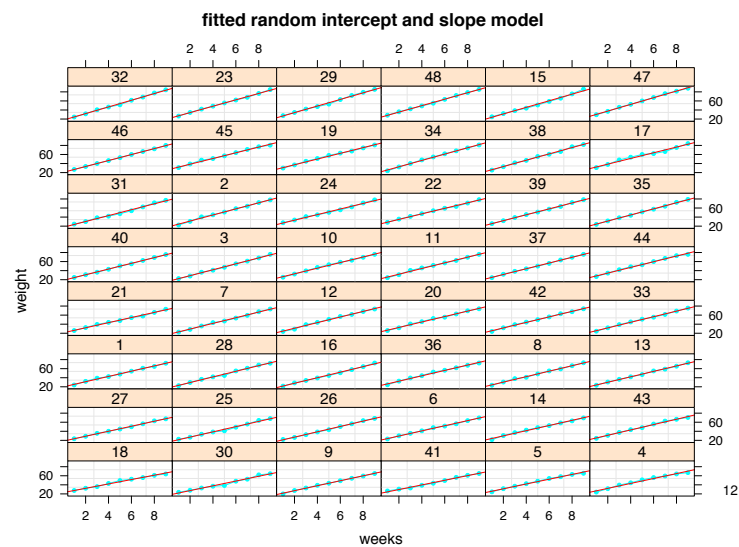
where

$$\begin{bmatrix} U_i \\ V_i \end{bmatrix} \underset{\text{ind.}}{\sim} N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_U^2 & \rho_{UV}\sigma_U\sigma_V \\ \rho_{UV}\sigma_U\sigma_V & \sigma_V^2 \end{bmatrix} \right)$$

are independent of the

$$\varepsilon_{ij} \underset{\text{ind.}}{\sim} N(0, \sigma_\varepsilon^2).$$

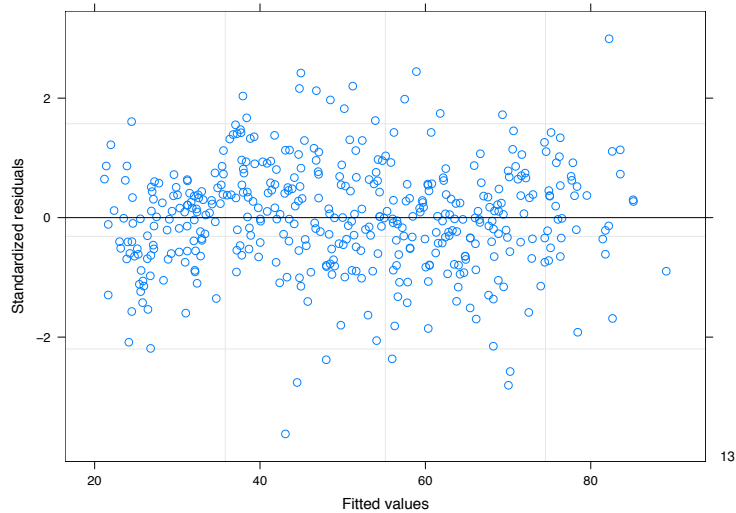
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## Fundamental Question

How are the **red lines** fitted?



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## Comments on Previous Two Slides

Close inspection of the fitted **red lines** shows that the lines have slightly different slopes

The **residual plot** is now showing **no systematic patterns**

The second model is an improvement.

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## Partial Answer

The fitted line for the  $i$ th pig is

$$\hat{U}_i + \hat{\beta}_0 + (\hat{\beta}_1 + \hat{V}_i) \text{ weeks}$$

The **fixed effects** estimates  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are obtained via **maximum likelihood**.

How about the **random effects** counterparts:  $\hat{U}_i$  and  $\hat{V}_i$ ?

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## Answer

Maximum likelihood not defined for random effects.

We need to appeal to best prediction theory:

$\hat{U}_i$  = best predictor of  $U_i$  given (weight) data

i.e.

$$\hat{U}_i = E(U_i | \mathbf{y})$$

where  $\mathbf{y}$  is the vector of weight observations.

Similarly for  $\hat{V}_i$ .

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## Whiteboard Interlude

We will now go through the mathematics of

**best prediction of random effects**

for general linear mixed models.

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