UNIVERSITY OF WOLLONGONG

School of Mathematics and Applied Statistics

STAT902. Advanced Data Analysis

ASSIGNMENT 2

Due: 5:00pm Monday 22nd March, 2010.
Local students: please put under lecturer's door.
Remote students: please post by this date.

Some of these questions use notation and results from the notes 'Likelihood Theory and Methods'; handed out in Class 2.

1. Let $\boldsymbol{A}$ be square matrix for which $\boldsymbol{I}+\boldsymbol{A}$ is invertible. Prove that

$$(\boldsymbol{I}+\boldsymbol{A})^{-1}=\boldsymbol{I}-\boldsymbol{A}(\boldsymbol{I}+\boldsymbol{A})^{-1}.$$

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the density

$$f_{X_i}(x;\theta)=\theta x^{\theta-1},\ 0<x<1;\ \theta>0.$$

   (a) Determine the maximum likelihood estimator of θ.

   (b) Derive an approximate 99% confidence interval for θ.

3. Let $X_1, \ldots, X_n$ be a random sample from the $N(\mu,\sigma^2)$ distribution. Let τ be the 0.95 quantile of the distribution, i.e. τ is defined by

$$P(X\le\tau)=0.95 \quad \text{where} \quad X\sim N(\mu,\sigma^2).$$

   (a) Find the maximum likelihood estimator for τ.

   (b) Derive an approximate 95% confidence interval for τ.

   (c) Suppose the data are:

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5.23  2.50  6.88  4.32  9.43  5.17
6.03  4.93  4.99  5.76  6.76  8.18
5.33  4.69  5.30  4.39  6.52  9.43
6.54  7.28  5.42  7.33  3.97  9.00
5.39
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   Find the maximum likelihood estimate of τ and obtain a corresponding standard error.

4. For $1\le i\le n$ let X_i given U_i be conditionally independent with probability function

$$P(X_i=x|U_i=u)=\binom{N}{x}u^x(1-u)^{N-x},\quad x=0,\ldots,N$$

where $U_1,\ldots,U_n$ are independent such that

$$f_{U_i}(u;\alpha,\beta)=\frac{u^{\alpha-1}(1-u)^{\beta-1}}{B(\alpha,\beta)},\quad 0<u<1.$$

Suppose that only the $X_1,\ldots,X_n$ are observed. The value of N is a fixed positive integer, but $\alpha>0$ and $\beta>0$ are to be estimated from the X_i's.

(a) Determine the likelihood function for (α, β).

(b) Determine the log-likelihood function for (α, β) and simplify as much as possible.

5. In *weighted* least squares fitting of the linear model

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

$\boldsymbol{\beta}$ is chosen to minimise

$$S(\beta; \boldsymbol{W}) = (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})^T \boldsymbol{W} (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})$$

where $\boldsymbol{W}$ is a diagonal square matrix with positive diagonal entries. Use vector calculus methods to obtain the solution for $\boldsymbol{\beta}$.