Appendix D

Polar co-ordinates

D.1 Transforming a planar system of differential equations from Cartesian co-ordinates to polar co-ordinates

Polar co-ordinates (r, θ) are defined in terms of Cartesian co-ordinates (x, y) by

$$\begin{aligned} x &= r\cos\theta, \\ y &= r\sin\theta. \end{aligned}$$

Recall that when the system of (Cartesian) differential equations

$$\begin{aligned} \dot{x} &= f(x, y), \\ \dot{y} &= g(x, y) \end{aligned}$$

is transformed into polar coordinates we obtain

$$\begin{aligned} r\dot{r} &= x\dot{x} + y\dot{y}, \\ r^2\dot{\theta} &= x\dot{y} - y\dot{x}. \end{aligned}$$

Question D.1 *Show that if*

$$\begin{aligned} x &= r\cos\theta, \\ y &= r\sin\theta. \end{aligned}$$

then

$$\begin{aligned} r\dot{r} &= x\dot{x} + y\dot{y}, \\ r^2\dot{\theta} &= x\dot{y} - y\dot{x}. \end{aligned}$$

D.2 Things to do

'Cartesian' coordinates. Footnote!