School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations
Assignment Week 12
Autumn 2008

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during the Tuesday lecture in Week 13

1. Consider the scheme

$$\frac{\mathrm{d}x}{\mathrm{d}t} - x^2 + \lambda y + \beta,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha x + \lambda^2 - y.$$

   (a) Find the defining conditions for a limit point bifurcation to occur in this system.

   (b) Draw the limit-point locus in the $\lambda - \alpha$ plane. Consider the cases: $\beta < 0$, $\beta = 0$ and $\beta > 0$.

   (c) From your answer to the previous question you will see that for a given value of β the number of limit points depends upon the value of α. Draw representative steady-state diagrams for all cases. For instance, when $\beta > 0$ the steady-state diagram has either one or three limit points and you should provide figures for both cases.

   (d) Calculate $\frac{\mathrm{d}^2\lambda}{\mathrm{d}x^2}$ for this system, showing all necessary working.

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 11
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Student Name: _____ *Student Number*: _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

2. An oxidation reaction in a CSTR is modelled by the system [1]

$$\frac{\mathrm{d}\mathcal{O}_2}{\mathrm{d}t} = 0.21q\,(1-\alpha) - 2q\mathcal{O}_2 - \frac{A_2\mathcal{P}}{T_0}\exp\left[-\frac{E_2}{T}\right]\mathcal{B}\mathcal{O}_2,$$
$$\frac{\mathrm{d}T}{\mathrm{d}t} = 2q\,(T_0 - T) + \frac{Q_2 A_2 \mathcal{P}^2}{T_0^2}\exp\left[-\frac{E_2}{T}\right]\mathcal{B}\mathcal{O}_2 - J\,(T - T_0)\,,$$

where the parameter $\mathcal{B}$ is given by

$$\mathcal{B} = \frac{\alpha A_1 - 0.21\,(1-\alpha)\,(2q + A_1)}{2\,(2q + A_1)} + \mathcal{O}_2.$$

(All variables and parameters have been scaled). In this question we take α as the primary bifurcation parameter ($0 \leq \alpha \leq 1$). The secondary bifurcation parameters are: $\mathcal{P}$ and T_0.

The parameter values are: $A_1 = 0.1$ $A_2 = 0.661 \times 10^9$; $E_2 = 46.308$; $\mathcal{P} = 4.4$; $Q_2 = 2828.074$, $T_0 = 1.5$; $J = 1$; $q = 0.00101$;

This system has two limit points. Find the values of α and the corresponding values of T^*.

You will need to make considerable use of Maple to solve this problem!

References

[1] M.I. Nelson and H.S. Sidhu. Bifurcation phenomena for an oxidation reaction in a continuously stirred tank reactor. II Diabatic operation. *The Anziam Journal*, 45:303–326, 2004.