

School of Mathematics & Applied Statistics
MATH971: Applied Non-Linear Differential Equations
Assignment Week 10
Autumn 2008

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during the Tuesday lecture in Week 11

1. Check that each of the following systems has an equilibrium point that exhibits the Hopf bifurcation at some value of α , and compute the first Lyapunov coefficient. Is the bifurcation subcritical or supercritical?

(a) *Rayleigh's equation*

$$\ddot{x} + \dot{x}^3 - 2\alpha\dot{x} + x = 0$$

(Hint: Introduce $y = \dot{x}$ and rewrite the equation as a system of two differential equations.)

(b) *Bautin's example*

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x + \alpha y + x^2 + xy + y^2;\end{aligned}$$

(c) *Brusselator:*

$$\begin{aligned}\dot{x} &= a - (\alpha + 1)x + x^2y \\ \dot{y} &= \alpha x - x^2y;\end{aligned}$$

2. For the planar system

$$\begin{aligned}x' &= 2(1 - \mu)x + x^2 - xy, \\ y' &= x + \frac{1}{2}x^2 - y,\end{aligned}$$

show that the critical point for which $y = x + \frac{1}{2}x^2$ and $x^2 = 4(1 - \mu)$ has a Hopf bifurcation at $x = 1$, $y = \frac{3}{2}$ and $\mu = \frac{3}{4}$. Determine if the bifurcation is subcritical or supercritical.

Hint. First make (and justify) the transformation $X = x - \epsilon$, $Y = y - \epsilon - \frac{1}{2}\epsilon^2$ and $\epsilon^2 = 4(1 - \mu)$.

3. In the Duffing equation

$$\ddot{x} + \mu\dot{x} + (x - x^3) = 0,$$

a bifurcation with pure imaginary eigenvalues occurs at $\mu = 0$ when the system changes from having negative to positive dissipation, but it is degenerate. Why?

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 11
Autumn 2008 Submission Receipt

Student Name: _____ *Student Number:* _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____