

School of Mathematics & Applied Statistics
MATH971: Applied Non-Linear Differential Equations
Assignment Week 9
Autumn 2008

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during the examples class in Week 10.

1. By using Dulac's test with $\rho = e^{-2x}$, show that

$$\begin{aligned}\frac{dx}{dt} &= y, \\ \frac{dy}{dt} &= -x - y + x^2 + y^2,\end{aligned}$$

has no limit cycles.

[Jordan & Smith]

2. Show that the following systems have no limit cycles:

[Jordan & Smith]

$$\begin{aligned}\text{(i)} \quad \frac{dx}{dt} &= -(1-x)^3 + xy^2 \quad \frac{dy}{dt} = y + y^3 \\ \text{(ii)} \quad \frac{dx}{dt} &= y \quad \frac{dy}{dt} = -1 - x^2, \\ \text{(iii)} \quad \frac{dx}{dt} &= 1 - x^3 + y^2 \quad \frac{dy}{dt} = 2xy;\end{aligned}$$

3. Show that the system

$$\begin{aligned}\frac{dx}{dt} &= \frac{y}{1+x^2} \\ \frac{dy}{dt} &= \frac{-x + y(1+x^2+x^4)}{1+x^2}\end{aligned}$$

has no limit cycle in \mathbb{R}^2 .

[Perko]

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 9
Autumn 2008 Submission Receipt

Student Name: _____ *Student Number:* _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____