School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations

Assignment Week 5

Autumn 2008

Student Name: ______________________________ *Student Number:* ______________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during the examples class in Week 7.

1. By determining the bifurcation diagram explain how the steady-state diagram for the singularity function

$$\mathcal{G} = x^3 - \lambda + \alpha x = 0$$

depends upon the value of α. Provide appropriate steady-state diagrams (do not show stability).

2. In this question we consider the singularity function

$$\mathcal{G} = x^2 - \lambda^2 + \alpha = 0$$

   (a) By determining the bifurcation diagram explain how the steady-state diagram for the singularity function depends upon the value of α. Provide appropriate steady-state diagrams (do not show stability), including the case $\alpha = 0$.

   (b) Is this an example of the 'transcritical singularity' or the 'isola singularity'?

3. In this question we consider the singularity function

$$\mathcal{G} = x^3 - \lambda x + \alpha + \beta x^2.$$

   (a) Obtain steady-state diagrams for regions A & D of figure 2.7. How many bifurcation points are there on there on each steady-state diagram?

   (b) Starting in region A of figure 2.7 explain how the steady-state diagram changes as you move from region A into region B, from region B into region C, from region C into region D and from region D into region A.

   (c) From figure 2.7 the point $(\alpha, \beta) = (0, 0)$ is special because the cusp singularity curve, $\alpha = \dfrac{\beta^3}{27}$ and the isola singularity curve, the line $\alpha = 0$, intersect at this point.

      (i) Find the steady-state diagram at the point $(\alpha, \beta) = (0, 0)$. (Do not determine stability)

      (ii) What kind of bifurcation occurs in this steady-state diagram?

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 5
Autumn 2008 Submission Receipt

Student Name: ______________________________ *Student Number:* ______________

Tutorial Class: __________ *Date Submitted:* ______________ *Tutor Initials:* ______________

In the following two questions $\mathcal{G}$ is a steady-state equation, x is a state variable, λ is the primary bifurcation parameter and α & β are secondary bifurcation parameters.

4. For each of the following functions construct the bifurcation diagram in the $\alpha - \beta$ plane and provide representative steady-state diagrams.

   (a) $\mathcal{F} = x^2 - \lambda^3 + \alpha + \beta\lambda = 0.$
   Show that there is a point on the isola locus where the degeneracy condition

   $$G_{xx}G_{\lambda\lambda} - G_{x\lambda}^2 \neq 0$$

   is violated. Mark the location of this point on your bifurcation diagram.

   (b) $\mathcal{F} = x^4 - \lambda + \alpha x + \beta x^2 = 0.$
   Show that there is a point on the cusp locus where the degeneracy condition

   $$G_{xxx} \neq 0$$

   is violated. Mark the location of this point on your bifurcation diagram.

5. The spruce budworm model is given by

   $$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{q}\right) - \frac{x^2}{1 + x^2}.$$

   Find all the steady-state diagrams for this problem when the primary bifurcation parameter is r. You may assume that $r > 0$ and $q > 0$.