School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations
Assignment Week 4
Autumn 2008

Student Name: ______________________ *Student Number:* ____________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during the examples class in Week 5.

1. Consider the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x^3 - \lambda x + \alpha + \beta x^2.$$

The singularity equation is

$$\mathcal{G} = x^3 - \lambda x + \alpha + \beta x^2 = 0.$$

(a) Figure 1 shows a steady-state diagram for the singularity equation

$$\mathcal{G} = x^3 - \lambda x + \alpha + \beta x^2 = 0.$$

(stability is not indicated).

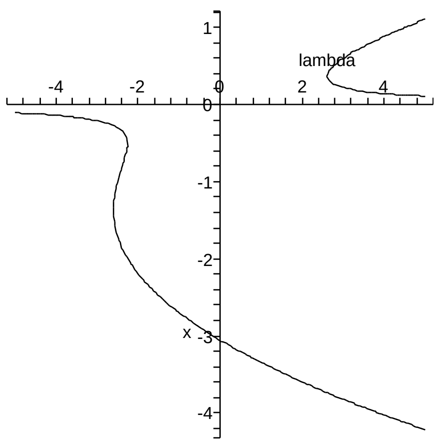

Figure 1: Steady-state diagram for the singularity equation $x^3 - \lambda x + \alpha + \beta x^2 = 0$. Parameter values: $\alpha = 0.5, \beta = 3.0$.

Write maple code to obtain this figure. Your answer must include your code and the graph that it generates.

(b) Figure 1 has three limit points.

(i) Using the Limit-point bifurcation theorem show that the bifurcation points of the singularity equation

$$\mathcal{G} = x^3 - \lambda x + 0.5 + 3x^2 = 0$$

are

$$(\lambda_1, x_1) = (-2.25, -0.5),$$
$$(\lambda_2, x_2) = (-2.60, -1.37),$$
$$(\lambda_3, x_3) = (2.60, 0.37).$$

(You may use maple as much as you require.)

(ii) Show that the required non-degeneracy conditions are satisfied.

(c) Hence write maple code to obtain figure 2. Your answer must include your code and the graph that it generates.

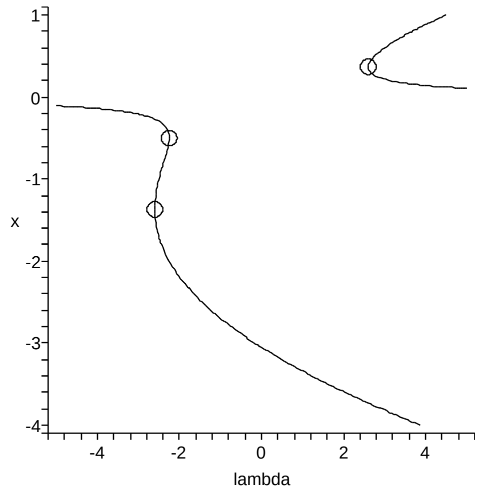

Figure 2: Steady-state diagram for the singularity equation $x^3 - \lambda x + \alpha + \beta x^2 = 0$. Bifurcation points marked by circles. Parameter values: $\alpha = 0.5, \beta = 3.0$.

(d) Determine the stability of the steady-state solutions shown on figure 2. Hence sketch a steady-state diagram that indicates stability.

2. The spruce budworm model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{q}\right) - \frac{x^2}{1+x^2}.$$

was investigated in the previous assignment. The steady-state solutions of this model were found to be $x_1 = 0$ and the solutions of the equation

$$\mathcal{G} = r\left(1+x^2\right)\left(1 - \frac{x}{q}\right) - x = 0.$$

Find the location of the limit points (r, x) for the singularity function $\mathcal{G}$ when

(i) $q = 10$.

(ii) $q = 20$.

Check that the appropriate non-degeneracy conditions are satisfied.

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations Assignment Week** **4**
Autumn 2008 Submission Receipt

Student Name: ____________________ *Student Number*: __________

Tutorial Class: ________ *Date Submitted:* __________ *Tutor Initials:* __________