

School of Mathematics & Applied Statistics
MATH971: Applied Non-Linear Differential Equations
Assignment Week 11
Autumn 2009

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during the examples class in Week 13.

The assignment that you hand in *must* include a cover page. On the cover-page you should briefly answer the following questions

- (a) What topic did you believe was the most important in the assignment?
- (b) Why do you believe that is the most important topic?
- (c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 13
Autumn 2009 Submission Receipt

Student Name: _____ *Student Number:* _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

1. In dimensionless form, the governing equations for the Belousov-Zhabotinskii reaction, as suggested by Tyson [1], are

$$\begin{aligned}\epsilon_1 \frac{dx}{d\tau} &= \frac{(q-x) fz}{q+x} + x(1-x), \\ \frac{dz}{d\tau} &= x - z.\end{aligned}\tag{1}$$

Here $x(\tau)$ and $z(\tau)$ represent the concentrations of $(HBr) O_2$ and $Ce(UV)$ respectively. The parameter ϵ_1 is a dimensionless combination of some of the rate constants, the quantity q is related to the concentration of a certain chemical reagent and f is a stoichiometric parameter in one of the reactions.

Consider the parameter values $f = 0.51391697$, $q = 0.004$. and $\epsilon_1 = 0.004$. Take the initial condition $(x, z) = (0, 0.4)$.

- (a) What is meant by the phrase ‘relaxation oscillation’?
- (b) (i) Integrate the equations for a long period of time until it appears that your solution has converged to a limit cycle. (Do not include this in your assignment).
(ii) Use the values of x and z at your last integration point as the initial conditions for a new integration. (Do not include this in your assignment).
(iii) Integrate the equations for several complete periods. (Do not include this in your assignment).
(iv) Plot $x(t)$, $z(t)$ and $x(t)$ against $z(t)$. Discuss your solution in the context of your answer to part (a) of this question.
2. Check that the following system has an equilibrium point that exhibits the Hopf bifurcation at some value of α . Compute the first Lyapunov coefficient and determine if the Hopf bifurcation is subcritical or supercritical.

$$\ddot{y} - (\alpha - y^2) \dot{y} + y = 0.$$

(Hint: Introduce $\dot{y} = x$ and rewrite the equation as a system of two differential equations.)

3. Check that each of the following systems has an equilibrium point that exhibits the Hopf bifurcation at some value of α , and compute the first Lyapunov coefficient. Is the bifurcation subcritical or supercritical?

- (a) *Rayleigh’s equation*

$$\ddot{x} + \dot{x}^3 - 2\alpha\dot{x} + x = 0$$

- (b) *Bautin’s example*

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x + \alpha y + x^2 + xy + y^2;\end{aligned}$$

- (c) *Brusselator*:

$$\begin{aligned}\dot{x} &= a - (\alpha + 1)x + x^2y \\ \dot{y} &= \alpha x - x^2y;\end{aligned}$$

References

- [1] J.J. Tyson. Oscillation, bistability and echo waves in model of the Belousov-Zhabotinskii reaction. *Ann. NY Acad. Sci.*, 316:279–95, 1979.