School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations

Assignment Week 9

Autumn 2009

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during the examples class in Week 11.

The assignment that you hand in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 11
Autumn 2009 Submission Receipt

Student Name: _____ *Student Number*: _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

1. A particular form of the *Holling-Tanner* model is given by

$$\dot{x} = f(x,y) = x\left(1 - \frac{x}{7}\right) - \frac{6xy}{7+7x},$$
$$\dot{y} = g(x,y) = 0.2y\left(1 - \frac{Ny}{x}\right).$$

In the previous assignment you showed that the non-trivial solution $\left(\bar{\mathbf{x}}_+, \frac{\bar{\mathbf{x}}_+}{N}\right)$ is unstable if $N_1 < N < N_2$.

Using the initial condition $(x,y) = (7,1)$ integrate the governing equations for three values of N in this range.

For your answer you should provide figures showing $x(t)$ as a function of time, $y(t)$ as a function of time and $y(t)$ against $x(t)$.

Using your figures what information can you extract about the behaviour of the solutions?

2. In dimensionless form, the governing equations for the Belousov-Zhabotinskii reaction, as suggested by Tyson [1], are

$$\begin{aligned} \epsilon_1 \frac{\mathrm{d}x}{\mathrm{d}\tau} &= \frac{(q-x)fz}{q+x} + x(1-x), \\ \frac{\mathrm{d}z}{\mathrm{d}\tau} &= x - z. \end{aligned} \tag{1}$$

Here $x(\tau)$ and $z(\tau)$ represent the concentrations of $(HBr)\,O_2$ and Ce(UV) respectively. The parameter ϵ_1 is a dimensionless combination of some of the rate constants, the quantity q is related to the concentration of a certain chemical reagent and f is a stoichiometric parameter in one of the reactions.

(a) Tell me about the Belousov-Zabotinskii reaction in one or two paragraphs. Provide references to books or web-pages you used as appropriate.

(b) Show that when $0 < q < 1$ that the region bounded by $q < x < 1$ and $0 < z < 1$ is positively invariant.

(c) Show that there is a critical value of ϵ, ϵ_{cr}, such that system (1) has no periodic solutions for $\epsilon > \epsilon_{\mathrm{cr}}$.

(d) By considering the Dulac function

$$\rho(x,z) = B_0 x^{-(1-\epsilon)}(1-x)^{-(k+1+\epsilon_1)} \exp\left[\frac{kz}{\epsilon_1}\right],$$

where k is an adjustable parameter, show that there is a critical value of the parameter f, f_{cr}, such that for $f > f\mathrm{cr}$ system (1) has no periodic solutions.

References

[1] J.J. Tyson. Oscillation, bistability and echo waves in model of the Belousov-Zhabotsinskii reaction. *Ann. NY Acad. Sci*, 316:279–95, 1979.