

School of Mathematics & Applied Statistics
MATH971: Applied Non-Linear Differential Equations
Assignment Week 7
Autumn 2009

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.

Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.

This assignment is to be handed in during the examples class in Week 9.

The assignment that you hand in *must* include a cover page. On the cover-page you should briefly answer the following questions

- (a) What topic did you believe was the most important in the assignment?
- (b) Why do you believe that is the most important topic?
- (c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

School of Mathematics & Applied Statistics **MATH971: Applied Non-Linear Differential Equations**
Assignment Week 9
Autumn 2009 Submission Receipt

Student Name: _____ *Student Number:* _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

1. A particular form of the *Holling-Tanner* model is given by

$$\begin{aligned}\dot{x} &= f(x, y) = x \left(1 - \frac{x}{7}\right) - \frac{6xy}{7+7x}, \\ \dot{y} &= g(x, y) = 0.2y \left(1 - \frac{Ny}{x}\right).\end{aligned}$$

We investigate the steady-state solutions of this system, and their stability, as a function of the parameter N ($N > 0$).

- (a) What does the Holling-Tanner model model?
 (b) In this question we find the physically meaningful steady-state solutions of the model.
 (i) Show that there are three steady-state solutions: $\bar{\mathbf{x}}_1$, in which the species y becomes extinct and $\bar{\mathbf{x}}_{\pm}$.
 (ii) Show that the steady-state solution $\bar{\mathbf{x}}_-$ is not of physical interest because both components of the steady-state solution are negative.
 (c) Determine the stability of the steady-state solution $\bar{\mathbf{x}}_1$.
 (d) In this question we investigate the stability of the steady-state solution $\bar{\mathbf{x}}_+$ as a function of the parameter N .
 (a) Show that along this solution branch we have

$$f_x(\bar{\mathbf{x}}_+) = \frac{2x^*(3-x^*)}{7(1+x^*)}.$$

(b) Show that

$$\det J = \frac{0.4x^*}{7(1+x^*)} \cdot \frac{1}{N} [Nx^* + 3(1-N^*)].$$

Hence, or otherwise, deduce that

$$\det J > 0 \forall N > 0.$$

(c) Show that

$$\begin{aligned}\text{tr } J &= \frac{2x^*(3^*)}{7(1+x^*)} - 0.2, \\ \text{tr } J = 0 &\Rightarrow x^{*2} - 2.3x^* + 0.7 = 0.\end{aligned}$$

- (d) Hence, or otherwise, deduce that there are critical values of N , N_1 and N_2 , such that the steady-state solution $\bar{\mathbf{x}}_+$ is
stable if $0 < N < N_1$
unstable if $N_1 < N < N_2$
stable if $N_2 > N$
 (e) Draw steady-state diagrams showing how the values for x and y vary as a function of N .

2. Use appropriate Liapunov functions to determine the stability of the steady state solutions of the following systems

- (a) $\dot{x}_1 = -x_1 + x_2 + x_1x_2$, $\dot{x}_2 = x_1 - x_2 - x_1^2 - x_2^3$,
 (b) $\dot{x}_1 = x_1 - 3x_2 + x_1^3$, $\dot{x}_2 = -x_1 + x_2 - x_2^2$,
 (c) $\dot{x}_1 = -x_1 - 2x_2 + x_1x_2^2$, $\dot{x}_2 = 3x_1 - 3x_2 + x_2^3$,
 (d) $\dot{x}_1 = -4x_2 + x_1^2$, $\dot{x}_2 = 4x_1 + 1 + x_2^2$.