School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations
Assignment Week 5
Autumn 2009

Student Name: ______________________________ *Student Number:* ____________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in at the end of the lecture in Week 7

The assignment that you hand in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

1. The spruce budworm model is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{q}\right) - \frac{x^2}{1+x^2}.$$

   The steady-state solutions of this model are $x_1 = 0$ and the solutions of the singularity equation

$$\mathcal{G} = r\left(1+x^2\right)\left(1 - \frac{x}{q}\right) - x = 0.$$

   In this equation r is the primary bifurcation parameter and q is a secondary bifurcation parameter.

   (a) Find the location of the limit points (r, x) for the singularity function $\mathcal{G}$ when

      (i) $q = 10$.

      (ii) $q = 20$.

      Check that the appropriate non-degeneracy conditions are satisfied.

      Hint. First use `implicitplot` to draw a steady-state diagram for the function $\mathcal{G}$ for the specified values of q.

   (b) For what value of q does the cusp singularity occur. (Check that the non-degeneracy conditions are satisfied).

   (c) Can the isola singularity occur in this model?

   (d) Can the double limit-point singularity occur in this model?

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Tutorial Class: ________ *Date Submitted:* ____________ *Tutor Initials:* ____________

3. Consider the singularity function

$$\mathcal{G} = x^2 + \lambda^4 + \alpha + \beta\lambda - 5\lambda^2, \tag{1}$$

where x is the state variable, λ is the primary bifurcation parameter and α & β are the secondary bifurcation parameters.

(a) Write down the conditions for an isola singularity to occur and solve . (Do not check the non-degeneracy conditions). Write the isola locus in the form $(\alpha(\lambda), \beta(\lambda))$.

(b) Identify the values of x, λ, α and β at any points where the non-degeneracy conditions for the isola singularity fail.

(c) Construct a bifurcation diagram for the singularity function in the $\alpha(x) - \beta(y)$ plane. Identify the location of any points on the bifurcation diagram where non-degeneracy conditions fail.

Remark. If the conditions for each singularity are expressed in the form $(\alpha(\lambda), \beta(\lambda))$ their locus can be drawn as a parametric plot.

(d) How many generic steady-state diagrams does equation (1) have?

Show an example of every generic steady-state diagram (plotting the state variable x as a function of the primary bifurcation parameter λ). For each steady-state diagram calculate the location of any limit point(s). Identify any limit point(s) on your steady-state diagram.

4. The Semenov model for the self-heating of a combustible material can be written in the form

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = \psi \exp\left[\frac{\theta}{1+\epsilon\theta}\right] - \theta,$$

where θ is essentially the temperature difference between the combustible material and its surroundings, τ is scaled time, ψ, $\psi > 0$, is known as the Semenov number and ϵ is a reduced activation energy. In practical problems $0 < \epsilon \ll 1$.

(a) In this part we consider the limiting case when $\epsilon = 0$.

(i) Determine the steady-state diagram (including stability).

(ii) For what values of (θ, ψ) does a limit-point bifurcation occur? Determine these values by applying the limit-point bifurcation theorem. (theorem 2.1)

(b) In this part we consider the case when $\epsilon > 0$.

(a) Determine the bifurcation diagram for this problem.

(b) Provide examples of all steady-state diagrams (including stability) for this problem.

(c) Show that the location of the limit points (should they exist) can be found by solving a quadratic equation in θ which is parameterised by ϵ but not ψ .