School of Mathematics & Applied Statistics

MATH971: Applied Non-Linear Differential Equations

Assignment Week <u>3</u>

Autumn 2009

Student Name: ______________________________ *Student Number:* ______________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in at the end of the lecture in Week <u>5</u>

The assignment that you hand in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

1. Consider the differential equation
$$x' = 2\mu + x^2$$
   (a) Find the steady-state solutions of this equation.
   (b) Determine the stability of the steady-state solutions as a function of the primary bifurcation parameter μ.
   (c) Draw a steady-state diagram for this differential equation.
   (d) For what value of the parameter μ does a bifurcation occur? What kind of bifurcation is it?

2. Consider the differential equation
$$x' = x\left(\mu - 3x + x^2\right)$$
   (a) Find the steady-state solutions of this equation.
   (b) Determine the stability of the steady-state solutions as a function of the primary bifurcation parameter μ.
   (c) Draw a steady-state diagram for this differential equation.
   (d) For what value of the parameter μ does a bifurcation occur? What kind of bifurcation is it?

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Tutorial Class: __________ *Date Submitted:* ______________ *Tutor Initials:* ______________

3. Consider the differential equation

$$x' = x\left(\mu + x - x^2\right)\left(1 - \mu + x^2\right)$$

   (a) Find the steady-state solutions of this equation.
   (b) Determine the stability of the steady-state solutions as a function of the primary bifurcation parameter μ.
   (c) Draw a steady-state diagram for this differential equation.
   (d) For what value of the parameter μ does a bifurcation occur? What kind of bifurcation is it?

4. The Semenov model for the self-heating of a combustible material can be written in the form

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau} = \psi \exp\left[\theta\right] - \theta,$$

   where θ is essentially the temperature difference between the combustible material and its surroundings, τ is scaled time and ψ is known as the Semenov number. Assume that $\psi = 0.3$.

   (a) By plotting $\frac{\mathrm{d}\theta}{\mathrm{d}\tau}$ as a function of θ show graphically that the model has two steady-state solutions, θ_1 and θ_2 with $0 < \theta_1 < \theta_2$.
   (b) Using your graph determine the stability of the steady-state solutions θ_1 and θ_2.
   (c) Using maple find the values of θ_1 and θ_2 to three decimal places.
   (d) Using maple find the eigenvalue, to two decimal places, of the steady-state solution θ_1 and the steady-state solution θ_2.