Family Name	
First Name	
Student Number	
Tutorial Day and Time	

University of Wollongong School of Mathematics and Applied Statistics MATH 151 — GENERAL MATHEMATICS 1A Autumn Session Class Test 3

Time Allowed: 50 minutes

This test consists of two parts.

Part A: 9 Questions. These questions are worth a total of 10 marks.

Part B: 6 Questions. These questions are worth a total of 10 marks.

Directions to Candidates

1. Answer questions in Part A and Part B in the space provided, showing full working.

Examination Materials/Aids Allowed

Non-alphanumeric, non-programmable, calculators are permitted. A one-page, double-sided, A4 size summary sheet is permitted.

This test paper is NOT to leave this room.

Part A: Short Answer Questions

Show full working to the following problems in the space provided. Questions 1–8 are worth one mark each. Question 9 is worth two marks.

1. Sketch the graph of $y = x^2$. Use this graph to sketch $y = \frac{1}{x^2}$.

2. A culture of a certain bacteria initially weighs 1 gram and doubles in size every 12 hours. What will be the weight of the sample after 2 days?

3. The formula for the radioactive decay of Polonium is given by $p(t) = e^{-0.005t}$ (t in days). A certain substance has $\frac{1}{3}$ of its original amount of polonium when tested. Find the approximate age of the substance (in days).

4. A logistic model p(t) is given by $p(t) = \frac{3}{1 + Ce^{-t}}$. If p(0) = 1, then find the value of C.

5. The magnitude, R, of an earthquake on the Richter scale, is given by $R = \log_{10} \left(\frac{a}{T}\right) + B$. If a certain receiving station measures R = 4 when $\frac{a}{T} = 1$, then find the value of R at this station when $\frac{a}{T} = 3$.

6. Evaluate the limit $\lim_{t\to\infty} \frac{3}{2+6e^{-2t}}$.

7. Evaluate the limit $\lim_{x \to 3} \frac{x-3}{x^2-9}$.

8. Let f(t) be the amount of rainfall (measured in cm) that has fallen since midnight, where t is the time (measured in hr). Interpret the following in practical terms, giving units: f(1) = 3.1.

9. Sketch the graph of the function $y = f(x) = -1 + 3\cos 2x$, noting its period. (This question is worth two marks)

Part B

Show full working to the following problems in the space provided. The number of marks for each question is shown.

1. Find the exponential function which best describes the following data.

[2]

2. When cancer cells are subjected to radiation treatment, the proportion of cells that survive the treatment (P) is given by

$$P = e^{-kr},$$

where r is the radiation level (in Roentgens) and k a constant. It is found that 30% of the cancer cells survive when r = 400 Roentgens. What should the radiation level be in order to allow 2% to survive? [2]

3. In an experiment on a sample of liquid a certain biological variable y is observed to reach its maximum 10 seconds after the start of the experiment and its minimum 70 seconds after the start of the experiment.

The behaviour is observed to vary in an approximately sinusoidal manner, oscillating between the values y = 0 and y = 4. Find a formula for y as a function of t. [2]

4. Are the following calculations correct? For each incorrect answer explain where the error is. [1]Q. The definition of pH is

$$pH = -\log\left[H^+\right],$$

where $[H^+]$ is the concentration of hydrogen ions measured in units of mol dm⁻³. Without using a calculator, what is the pH of a solution containing a concentration of hydrogen ions of 10^{-5} mol dm⁻³? Simplify as far as possible.

A1.

$$pH = -\log [H]^+,$$
$$\Rightarrow pH = -e^{10^{-5}}$$

A2.

$$pH = -\log [H]^+,$$

$$\Rightarrow pH = -\log 10^{-5}.$$

5. Are the following calculations correct? For each incorrect answer explain where the error is. ${\bf Q}.$ The definition of pH is [+++]

$$pH = -\log\left[H^+\right],$$

where $[H^+]$ is the concentration of hydrogen ions measured in units of moldm⁻³. The pH of a solution of trifluoroethanoic acid is 4.5. What is the concentration of hydrogen ions? $\mathbf{A1}$

$$\begin{split} 4.5 &= -\log\left[\mathrm{H}^+\right],\\ \Rightarrow 10^{4.5} &= \left[\mathrm{H}^+\right], \end{split}$$

 $\mathbf{A2}$

$$\begin{split} 4.5 &= -\log\left[\mathrm{H}^+\right],\\ \Rightarrow \log 4.5 &= \left[\mathrm{H}^+\right],\\ \Rightarrow \left[\mathrm{H}^+\right] &= 0.653. \end{split}$$

6. A culture of bacteria initially weighs 0.15 g and its weight doubles every 45 minutes. What will be the weight have 100 minutes? [2]

[1]