MATH141 – Autumn 2008
Tutorial Sheet – Week 4

Solutions available as of Friday at the MATH141 web site:
`http://www.uow.edu.au/~mnelson/teaching.dir/math141.html`

1. Simplify $\sum_{k=1}^{m} c_{ik}\delta_{kj}$, where i, j, k are integers, when
   (a) $1 \leq j \leq m$, (b) $j > m$.
2. Find the value(s) of k so that $2x^3 - 3x^2 - kx + 20$ is divisible by $x - 5$.
3. (a) Evaluate $\sum_{k=0}^{k=n} \alpha r^k - r \sum_{k=0}^{n} \alpha r^k$.
   (b) Use the result in (a) to prove that
$$\begin{aligned}\sum_{k=0}^{n} \alpha r^k &= \alpha + \alpha r + \alpha r^2 + \cdots + \alpha r^n \\ &= \frac{\alpha - \alpha r^{n+1}}{1-r}, \quad (r \neq 1).\end{aligned}$$
4. Determine $\left(12x^3 - 11x^2 - 25\right) \div (3x - 5)$.
5. Given
$$A = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix},$$
   compute the following.
   (a) $(AB)^T$ (b) $B^T A^T$ (c) $A^T B^T$
   What conclusions do you draw?
6. Test whether $(x - 2)$ is a factor of $f(x) = 2x^3 + 2x^2 - 17x + 10$. If so, factorise as far as possible.
7. Compute ABC where
$$A = (2 - 1), \quad B = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 6 \\ -5 \end{pmatrix}.$$
8. Write down the complete binomial expansion of the following.
   (a) $(2 - x)^5$ (b) $(y + 3)^7$
9. Find the set of value that satisfy $\left|\frac{3x+2}{5}\right| < 4$.
10. By taking logarithms to base three convert the equation
$$3^x 8^{\frac{x}{x+2}} = k$$
    to the quadratic equation
$$x^2 + \left[\log_3\left(\frac{72}{k}\right)\right] x - \log_3\left(k^2\right) = 0.$$
    Solve this equation for the cases
    (a) $k = 1$.
    (b) $k = 6$.

Note. Part (b) is optional — it is *much much* harder then any question you will be asked on the MST/exam. Hint *Before* you start part (b) solve the equation $x^2 + ax - (a+1) = 0$.

Source. *The College Mathematics Journals*, **37**(1), page 42, January 2006.

Week 4 Lecture Material

FUNDAMENTALS	**ALGEBRA**
(Mark Nelson)	(Tim Marchant)
Sections 1.13 & 2.1	Read upto Section 5.12
Exercises 1.13.7, 2.1.4	**Exercise 5C**