MATH141 – Autumn 2008
Tutorial Sheet – Week 4

Solutions available as of Friday at the MATH141 web site:

1. Simplify \( \sum_{k=1}^{m} c_{ik} \delta_{kj} \), where \( i, j, k \) are integers, when
   (a) \( 1 \leq j \leq m \),
   (b) \( j > m \).

2. Find the value(s) of \( k \) so that \( 2x^3 - 3x^2 - kx + 20 \) is divisible by \( x - 5 \).

3. (a) Evaluate \( \sum_{k=0}^{n} \alpha r^k \).
   (b) Use the result in (a) to prove that
   \[
   \sum_{k=0}^{n} \alpha r^k = \frac{\alpha - \alpha r^{n+1}}{1 - r}, \quad (r \neq 1).
   \]

4. Determine \( (12x^3 - 11x^2 - 25) \div (3x - 5) \).

5. Given
   \[
   A = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix},
   \]
   compute the following.
   (a) \( (AB)^T \)
   (b) \( B^T A^T \)
   (c) \( A^T B^T \)
   What conclusions do you draw?

6. Test whether \( (x - 2) \) is a factor of \( f(x) = 2x^3 + 2x^2 - 17x + 10 \). If so, factorise as far as possible.

7. Compute \( ABC \) where
   \[
   A = (2 - 1), \quad B = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 6 \\ -5 \end{pmatrix}.
   \]

8. Write down the complete binomial expansion of the following.
   (a) \( (2 - x)^5 \)
   (b) \( (y + 3)^7 \)

9. Find the set of value that satisfy \( \left| \frac{3x + 2}{5} \right| < 4 \).

10. By taking logarithms to base three convert the equation
    \[ 3^2 8^{1/5} = k \]
    to the quadratic equation
    \[ x^2 + \log_3 \left( \frac{72}{k^2} \right) x - \log_3 (k^2) = 0. \]
    Solve this equation for the cases
    (a) \( k = 1 \).
    (b) \( k = 6 \).


Note. Part (b) is optional — it is much much harder then any question you will be asked on the MST/exam. Hint Before you start part (b) solve the equation \(x^2 + ax - (a + 1) = 0\).


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**Week 4 Lecture Material**

**FUNDAMENTALS**

(Mark Nelson) Sections 1.13 & 2.1

**ALGEBRA**

(Tim Marchant) Read upto Section 5.12

**Exercises 1.13.7, 2.1.4**

**Exercise 5C**