# Fundamentals Lecture Three

**Logarithms** Simplify

$$\log_a 18 - 2\log_a 3$$

**Factorisation** Simplify

$$2x^2 - 5x - 12$$

**Algebraic Fractions** Simplify

$$\frac{m^2+m-2}{m^2-m}$$

# Logarithms

Consider a simple example,

$$16 = 2^4.$$

2 is called the *base* of the number, and 4 is called the *index* or the *logarithm to base 2 of 16.*

$$\begin{aligned} 16 &= 2^4, \\ \text{number} &= \text{base}^{\text{Index}}. \\ \text{OR} \quad 4 &= \log_2 16, \\ \text{logarithm} &= \log_{\text{base}} \text{number}. \end{aligned}$$

More formally. If

$$N = b^x \Leftrightarrow x = \log_b N,$$

where $b$ and $N$ are positive real numbers. then $x$ is called the *logarithm to base b of* $N$.

# Rules

Let $x$ and $y$ be positive real numbers.

1. $\log_b xy = \log_b x + \log_b y$
2. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
3. $\log_b x^p = p \log_b x$
4. $\log_a a = 1$ (because $a^1 = a$)
5. $\log_a 1 = 0$ (because $a^0 = 1$)

**Note**: There is *no* rule of logarithms for simplifying either $\log(x+y)$ or $\log(x-y)$

**Common Logarithms** are logarithms to the base 10, and are written either as $\log_{10} N$ or, simply, $\log N$.

**Natural Logarithms** are logarithms to the base $e$, and are written either as $\log_e N$ or, most frequently, as $\ln N$.

**Note**:

(i) $\ln e = 1.$

(ii) $\ln e^x = x,$

(iii) $e^{\ln x} = x,$

Simplify the following expressions

1. $\log_2 16$
2. $\log_5 50 + \log_5 10 - \log_5 4$
3. $\dfrac{\log 32}{\log 8}$
4. $4 \log 3 - \log 27$

# Exercises on Logarithms

Exercise 1.3.2. (page 7)

For each of the questions do as many of the subquestions as you require in order to gain mastery of the basic technique.

$$(4)$$

$$(3)$$

$$\left(\frac{5}{3}\right)$$

$$(\log 3)$$

# Factorisation

*Factorisation* of an algebraic expression consists of rewriting the given algebraic expression as a *product*. The terms in the product are called *factors*.

**Recall**

$$x^2 + 2xy + y^2 = (x + y)^2$$
$$x^2 - 2xy + y^2 = (x - y)^2$$

# Common Factors

Factorise

$$ab + ac + 3a =$$

# Common Factors by Grouping

Factorise the following expressions

1. $ax + bx + ay + by$
2. $x^2 - y^2 - 6x + 6y$
3. $x^2 - 5x + 6 - ax + 2a$

# Difference of Two Squares

Factorise the following expressions

1. $x^2 - y^2$
2. $a^4 - b^4$
3. $(2x + 3y)^2 - (x - 4y)^2$

$$((a+b)(x+y))$$

$$((x-y)(x+y-6))$$

$$((x-2)(x-3-a))$$

$$((x-y)(x+y))$$

$$((a-b)(a+b)\left(a^2+b^2\right))$$

$$((3x-y)(x+7y))$$

# Quadratic Factors

Factorise the following expressions

1. $x^2 + (a+b)\,x + ab$

2. $x^2 - 9x + 20$

3. $6x^2 + 11x + 3$

# Exercises on Factorisation

Exercise 1.4.1 (page 9)

Do as many of the questions as you require in order to gain mastery of the technique.

$$(x+a)(x+b)$$

$$(x-5)(x-4)$$

$$(2x+3)(3x+1)$$

# Algebraic Fractions Rules

1. $\frac{am}{bm} = \frac{a}{b}$
2. $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c} \ (c \neq 0)$
3. $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd} \ (b, d \neq 0)$
4. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \ (b, d \neq 0)$
5. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \ (b, c, d \neq 0)$

## Note

1. In $\frac{a+m}{b+m}$ the '$m$' *cannot* be cancelled.
2. $\frac{a}{\frac{b}{c}} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$
3. $\frac{\frac{a}{b}}{c} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$

# Examples

Simplify the following expressions

1. $\frac{(x-2)}{(x+2)} \times \frac{3x+6}{x^2+2x-8}$
2. $\frac{2}{a-3} \div \frac{6}{a^2-9}$
3. $\frac{3}{b^2-4} - \frac{5}{3b-6}$

$$\left(\frac{3}{x+4}\right)$$

$$\left(\frac{a+3}{3}\right)$$

$$\left(\frac{-5b-1}{3(b-2)(b+2)}\right)$$

# Exercises on Algebraic Expressions

Exercise 1.5.2 (page 11)

Do as many of the questions as you require in order to gain mastery of the technique.