Fundamentals Lecture Two

Indices Simplify

$$6x^3y^{-2} \times \frac{1}{24}x^{-5}y^4$$

Surds Simplify

1. $\dfrac{\sqrt{5}}{\sqrt{45}}$
2. $\dfrac{5}{1+\sqrt{5}}$

Definition

If a is a real number and n is a natural number, then

$$a^n = a \times a \times \ldots \times a \quad (n \quad \text{factors}).$$

n is called either the *index* of a, the *exponent* on a, or the *power* to which a is raised. a is called the *base*. □

Rules

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(ab)^m = a^m b^m$
4. $(a^m)^n = a^{mn}$
5. $a^{-m} = \frac{1}{a^m}$
6. $a^0 = 1$
7. $a^{m/n} = (^n\sqrt{a})^m =^n \sqrt{a^m}$

Examples

Simplify the following expressions

1. $$\frac{3xy^2 \times 4x^3 \times y}{2xy \times 4y^2}$$

2. $$\frac{6x^{-4} \times 2x^3}{3x^{-3}}$$

3. $\left(5x^2y^{-3/2}z^{1/4}\right)^2 \times \left(4x^4y^2z\right)^{-1/2}$

$$\left(\frac{3}{2}x^3\right)$$

$$\left(4x^2\right)$$

$$\left(\frac{25}{2}x^2y^{-4}\right)$$

Definition

A *surd* is the name given to an *irrational* number that can be expressed in the form $\sqrt[n]{a}$.

(See definition of irrational p1-1)

Notation

$\sqrt{a}$, $\surd a$ or $a^{1/2}$ means "square root of a"

$\sqrt[n]{a}$, $^n\surd a$ or $a^{1/n}$ means "n^{th} root of a"

Irrational numbers such as $\sqrt{2}$, $5\sqrt{3}$, $\sqrt[3]{7}\ldots$ are *surds*, whereas $\sqrt{4}$, $\sqrt[3]{27}, 3\sqrt{9}\ldots$ are *not*, since they can be evaluated *exactly*.

Question. Evaluate exactly $\sqrt{4}$, $\sqrt[3]{27}\,\&\,3\sqrt{9}$.

Rules

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$
3. $(\sqrt{x})^2 = x$ provided $x \geq 0$
4. $\sqrt{x^2} = \begin{cases} +x & \text{if } x \geq 0, \\ -x & \text{if } x < 0. \end{cases}$
5. $a\sqrt{x} \pm b\sqrt{x} = (a \pm b)\sqrt{x}$

Examples

Simplify the following expressions

1. $4\sqrt{3} + 5\sqrt{7} - 2\sqrt{3} + \sqrt{7}$
2. $2\sqrt{3} \times 4\sqrt{5}$
3. $\sqrt{18} \div \sqrt{2}$
4. $6\sqrt{15} \div 2\sqrt{3}$
5. $\sqrt{8}$
6. $\sqrt{3} \times \sqrt{21}$
7. $5\sqrt{18}$
8. $\sqrt{20} + \sqrt{45}$
9. $\sqrt{12} - \sqrt{27} + \sqrt{75} - \sqrt{15}$

$$\left(2\sqrt{3}+6\sqrt{7}\right)$$

$$\left(8\sqrt{15}\right)$$

$$(\pm 3)$$

$$\left(3\sqrt{5}\right)$$

$$\left(2\sqrt{2}\right)$$

$$\left(3\sqrt{7}\right)$$

$$\left(15\sqrt{2}\right)$$

$$\left(5\sqrt{5}\right)$$

$$\left(\sqrt{3}\left[4-\sqrt{5}\right]\right)$$

Rationalising the Denominator

The *conjugate* of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$. Multiplying a surd by its conjugate always gives a rational number

$$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$$
$$= \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2,$$
$$= a - b.$$

Using this rule we can express any surd with a rational denominator.

Examples

Simplify the following expressions.

1. $\dfrac{1}{\sqrt{a}}$

2. $\dfrac{1}{\sqrt{a}+\sqrt{b}}$

3. $\dfrac{3}{\sqrt{5}}$

4. $\dfrac{2}{6-\sqrt{3}}$

5. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$\left(\frac{\sqrt{a}}{a}\right)$$

$$\left(\frac{\sqrt{a}-\sqrt{b}}{a-b}\right)$$

$$\left(\frac{3\sqrt{5}}{5}\right)$$

$$\left(\frac{12+2\sqrt{3}}{33}\right)$$

$$\left(5+2\sqrt{6}\right)$$

Equality of surds

If $a+\sqrt{b}=c+\sqrt{d}$ where a and c are rational and $\sqrt{b}$ and $\sqrt{d}$ are surds, then $a=c$ and $b=d$.

Examples

Find the values of α and β in the following expressions

1. $\alpha+\sqrt{\beta}=2+\sqrt{5}$
2. $2\alpha+3\sqrt{\beta}=4+3\sqrt{2}$
3. $\alpha\sqrt{3}=\sqrt{27}$
4. $\alpha+2\sqrt{\beta}=\left(\sqrt{3}+\sqrt{5}\right)^2$

$(\alpha = 2, \beta = 5)$

$(\alpha = 2, \beta = 2)$

$(\alpha = 3)$

$(\alpha = 8, \beta = 15)$

Exercises on Indices and Surds

Exercise 1.2.4 (pages 4& 5)

For each of the questions do as many of the subquestions as you require in order to gain mastery of the basic technique.

DO NOT do question 4 on the surds worksheet.