

# Fundamentals Lecture Four

**Function Notation** What is a function?

**Zeros of Functions**  $x$  is a zero of the function  $f$  if  $f(x) = 0$ .

**Quadratic Equations**  $ax^2 + bx + c$

- Finding roots of quadratic equations.
- Equations that are reducible to quadratics.  $4^z - 9(2^z) + 8 = 0$ .
- Sketching quadratic equations.

# Function Notation

## A simple definition

A *function* is a mathematical rule that assigns to each and every input a *unique* output.

Examples of functions are

$$f(x) = \sin^2 x + 5x + 2$$

$$h(t) = e^{t^2} - 47$$

$$H(z) = \ln z + \frac{3}{1+z}$$

**Note** The ‘name’ of the variable is irrelevant. The rule for  $f(x)$  could be written as

$$f(w) = \sin^2 w + 5w + 2 \text{ or}$$

$f(t) = \sin^2 t + 5t + 2$ . It is the rule that is important.

## Example

If  $g(t) = \frac{3t+2}{4t-1}$  find

$$g(1)$$

$$g(0.5)$$

$$g(-t)$$

$$g(a+1).$$

## Solution

$$g(1) = \qquad =$$

$$g\left(\frac{1}{2}\right) = \qquad =$$

$$g(-t) = \qquad =$$

$$g(a+1) =$$

$$=$$

$$=$$

$$\left(\frac{5}{3}\right)$$

$$\left(\frac{7}{2}\right)$$

$$\left(\frac{-3t+2}{-4t-1}\right)$$

$$\left(\frac{3a+5}{4a+3}\right)$$

## Example

If  $g(x) = 5$  find

$$g(1) = \quad =$$

$$g(-3) = \quad =$$

$$g(-x) = \quad =$$

$$g(h+t) =$$

$$=$$

$$=$$

## Exercises on Function Notation

Exercise 1.6.2 (page 12)

# Zeros of Functions

## Definition

A *zero* of the function  $f(x)$  is a *solution* to the equation  $f(x) = 0$ .  $\square$

**Note** : A zero of the function  $f(x)$  can also be referred to as a *root* of the equation  $f(x) = 0$ .

For example, the zeroes (or roots) of  $f(x) = x^2 + 5x - 14$  can be found by solving the equation  $f(x) = 0$

$$\text{i.e. } x^2 + 5x - 14 = 0$$

$$\text{Factorising} \quad \quad \quad = 0$$

$$\text{i.e. } x = \underline{\quad} \text{ and } x = \underline{\quad}$$

So,  $x = \underline{\quad}$  and  $x = \underline{\quad}$  are the zeroes (or roots) of  $f(x) = x^2 + 5x - 14$ .

$$(x - 2)(x + 7)$$

# Exercises on Functions

Exercise 1.7.2 (page 13)

# Quadratic Equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

## 1. Solution by factors

### Example

Find the roots of the equation

$$3x^2 - 13x - 10 = 0$$

### Solution

$$0 = 3x^2 - 13x - 10$$

$$0 = ( \quad ) ( \quad )$$

$$\therefore \quad = 0 \text{ or } \quad = 0$$

$$\therefore x = \quad \text{or } x =$$

Thus the roots are  $\quad$  and  $\quad$

## 2. Quadratic Formula

The roots of the quadratic equations  $ax^2 + bx + c = 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta = b^2 - 4ac$  is called the *discriminant* of the quadratic equation.

If  $\Delta > 0$ : roots are real and unequal

If  $\Delta = 0$ : roots are real and equal

If  $\Delta < 0$ : no real roots

## 3. Completing the Square

Not needed.

## Example

Solve the equation

$$2x^2 - 3x - 4 = 0$$

## Solution

$$a = \quad , b = \quad , c =$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x =$$

$$=$$

$$=$$

# Equations Reducible to Quadratics

Many harder equations can be reduced to quadratic equations by use of a simple substitution.

## Examples

Solve

1.  $4x^4 + 7x^2 - 15 = 0$

2.  $4^x - 9(2^x) + 8 = 0$

Solve  $4x^4 + 7x^2 - 15 = 0$ . Let  $u = x^2$   
and equation becomes

$$0 =$$

$$\therefore 0 =$$

$$\therefore u = \quad \text{or } u =$$

But  $u = x^2$ , therefore

$$x^2 = \quad \text{OR} \quad x^2 =$$

$$\therefore x = \quad \text{No real sols}$$

$$\frac{5}{4} - 3$$

$$\frac{5}{4} - 3 \pm \frac{\sqrt{5}}{2}$$

Solve  $4^x - 9(2^x) + 8 = 0$ . Let  $u = 2^x$   
and equation becomes

$$0 =$$

$$=$$

$$\therefore u = \quad \text{OR} \quad u =$$

But  $u = 2^x$

$$\therefore 2^x = \quad \text{OR} \quad 2^x =$$

$$=$$

$$=$$

$$\therefore x = \quad \text{OR} \quad x =$$

Exercise 1.8.3  
(pages 15 & 16)

$$(u - 8)(u - 1)$$

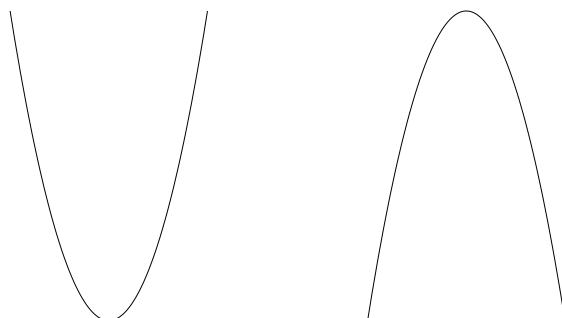
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# Mark's Guide to: *Sketching Quadratic Functions*

$$y = ax^2 + bx + c$$

1. Ignoring the placement of the axis the graph looks like one of these



depending upon if  $a > 0$  or  $a < 0$ .

2. Note that if  $a > 0$  the curve has a *minimum* whilst if  $a < 0$  it has a *maximum*.

$$y = ax^2 + bx + c$$

3. The position of the maximum/minimum is at  $x = \frac{-b}{2a}$ .  
The  $y$  value is  $\frac{4ac-b^2}{4a}$ .
4. Notice that the curve is *symmetric* around the maximum/minimum.
5. Find the values of  $x$  at which  $y = 0$ , i.e.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If  $b^2 - 4ac < 0$  then there are no (real) roots and the graph does not cross the  $y$ -axis.

6. Calculate the value of  $y$  when  $x = 0$ .

**Example** Sketch  $y = x^2 + 2x + 3$ .

## Solution

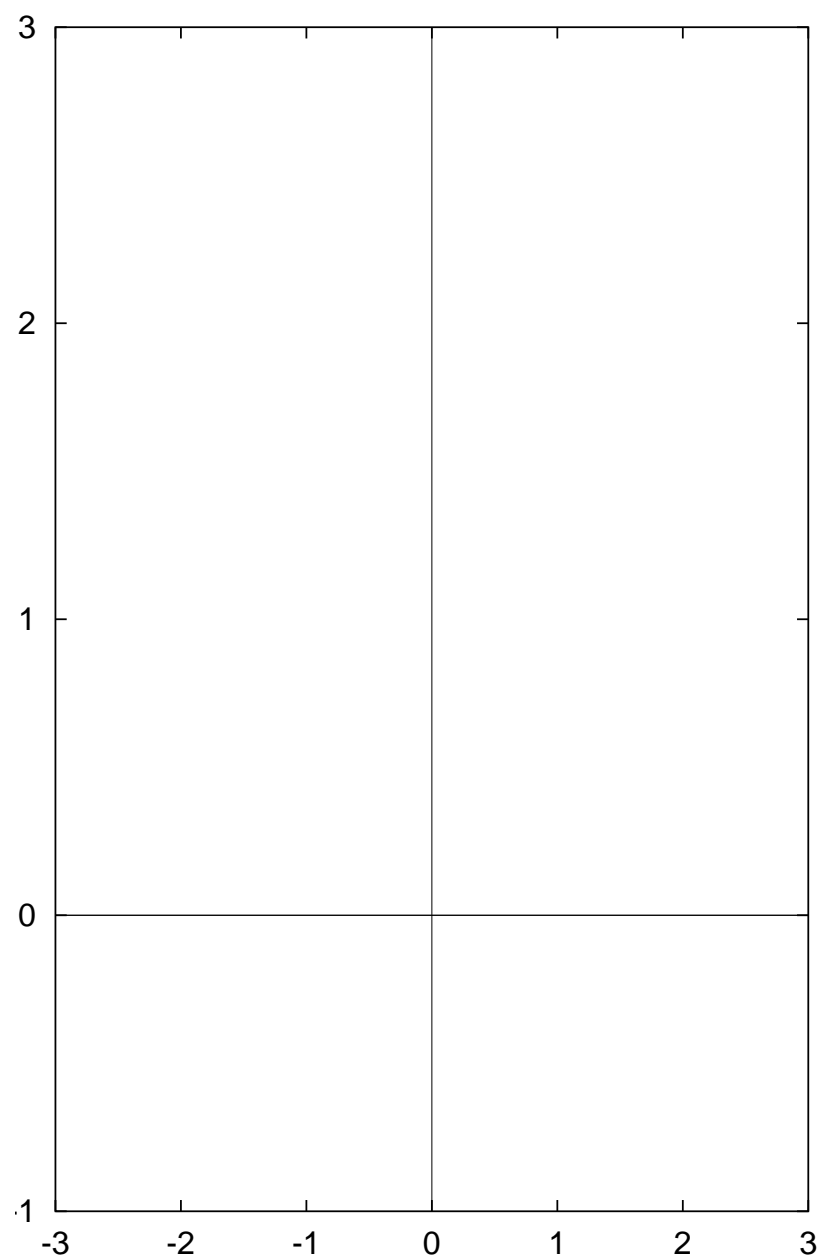
1.  $a$  is positive/negative therefore  $y$  has a maximum/minimum.
2. The location of the max/min is

$$x =$$

$$y =$$

3. What is the value of  $y$  when  $x = 0$ ?
4. Does the quadratic have real roots?  
If yes, where are they? What are the corresponding values for  $y$ ?

Graph of  $y = x^2 + 2x + 3$



**Example** Sketch  $y = -2x^2 + 5x - 2$

## Solution

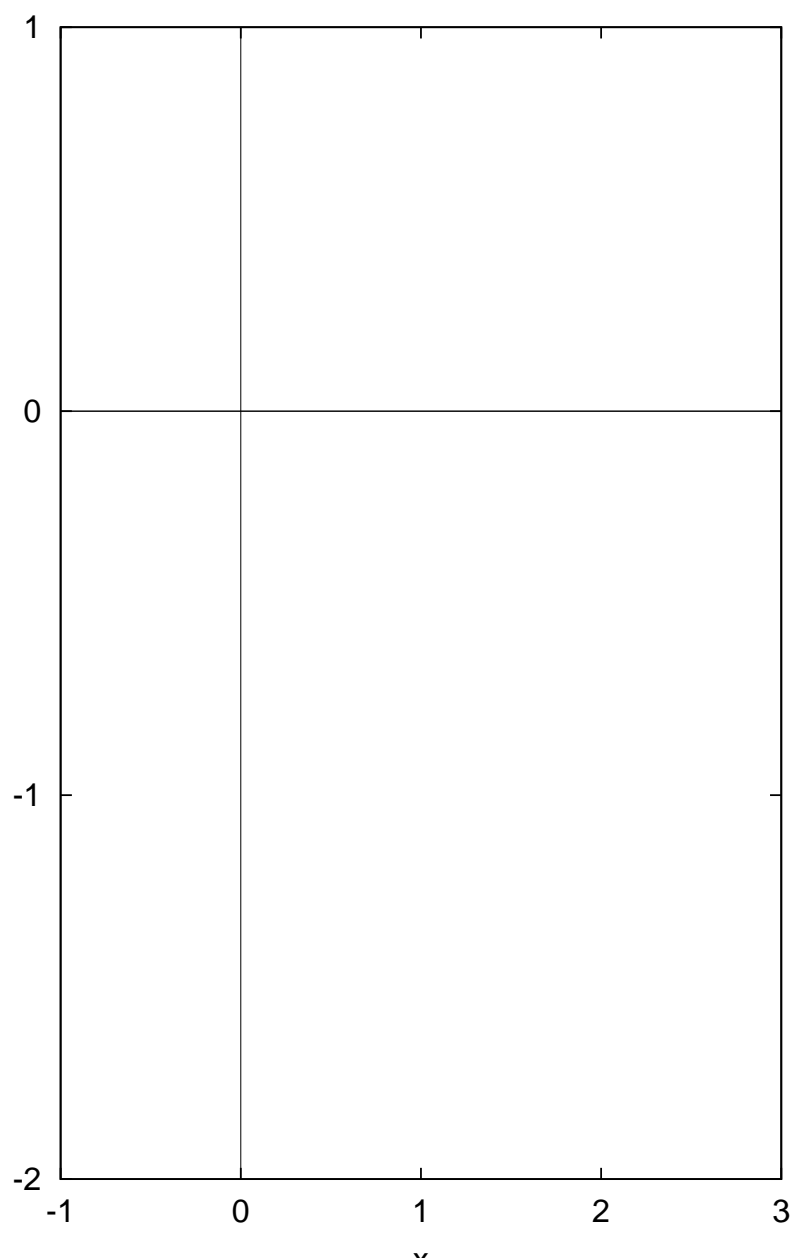
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4. Does the quadratic have real roots?  
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Graph of  $y = -2x^2 + 5x - 2$



# Exercises on Quadratic functions

Exercise 1I (page 1-19)

Q1. parts (b) and (c) & Q2.

Do as many of the questions as you require in order to gain mastery of the technique.