

Find an exact value for $\sin \frac{\pi}{12}$

Solution

$$\frac{\sqrt{2}}{4} (\sqrt{3} - 1)$$

$$\text{Now, } \frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\begin{aligned}\therefore \sin \frac{\pi}{12} &= \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\ &= \sin \left(\frac{\pi}{3} \right) \cos \left(\frac{\pi}{4} \right) - \cos \left(\frac{\pi}{3} \right) \sin \left(\frac{\pi}{4} \right) \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1}{2\sqrt{2}} (\sqrt{3} - 1) \\ &= \frac{\sqrt{2}}{4} (\sqrt{3} - 1)\end{aligned}$$

Use the sum of angles formulae and the double angle formulae to show that

$$\sin 3\omega = 3 \sin \omega - 4 \sin^3 \omega$$

$$\sin 3\omega = \sin (\omega + 2\omega)$$

$$= \sin \omega \cos 2\omega + \cos \omega \sin 2\omega$$

using the sum of angles formulae for $\sin A + B$.

$$= \sin \omega (1 - 2 \sin^2 \omega) + \cos \omega (2 \sin \omega \cos \omega)$$

using the double angle formulae for $\cos 2\omega$ and $\sin 2\omega$.

$$= \sin \omega - 2 \sin^3 \omega + 2 \sin \omega \cos^2 \omega$$

$$= \sin \omega - 2 \sin^3 \omega + 2 \sin \omega (1 - \sin^2 \omega)$$

using Pythaoreas formula

$$= 3 \sin \omega - 4 \sin^3 \omega$$