

2.9 LOGARITHMIC DIFFERENTIATION

We use **Logarithmic Differentiation** when we have a function of the form $y = g(x)^{h(x)}$. (Note that $g(x)$ may be a constant.) For example

$$y = 2^x, \quad y = x^x, \quad y = x^{\tan x}.$$

Recall

1. $e^{\ln a} = a, \quad a > 0$
2. $\ln a^b = b \ln a$
3. $\frac{d}{dx} [e^{f(x)}] = e^{f(x)} f'(x)$

Example

$$\begin{aligned} \frac{d}{dx} [b^x] &= \frac{d}{dx} [e^{\ln b^x}] && \text{(using (i))} \\ &= \frac{d}{dx} [e^{x \ln b}] && \text{(using (ii))} \\ &= \frac{e^{x \ln b} \frac{d}{dx} (x \ln b)}{\phantom{e^{x \ln b} \frac{d}{dx} (x \ln b)}} && \text{(using (iii))} \\ &= \frac{e^{x \ln b} \cdot \ln b}{\phantom{e^{x \ln b} \cdot \ln b}} \\ &= \underline{b^x \ln b} \end{aligned}$$

Example

$$\begin{aligned}
\frac{d}{dx} [x^{\sin x}] &= \frac{d}{dx} [e^{\ln x^{\sin x}}] \text{ (using (i))} \\
&= \frac{d}{dx} [e^{\sin x \ln x}] \text{ (using (ii))} \\
&= \frac{e^{\sin x \ln x} \cdot \frac{d}{dx} (\sin x \ln x)}{\phantom{e^{\sin x \ln x}}} \\
&\text{(using (iii))} \\
&= \frac{e^{\sin x \ln x} \left(\cos x \ln x + \sin x \cdot \frac{1}{x} \right)}{\phantom{e^{\sin x \ln x}}} \\
&= \frac{x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right)}{\phantom{x^{\sin x}}}
\end{aligned}$$

2.9.1 Questions

Do them!