Essential Mathematics Skills - Question Database - Chapter 4

Steven I. Barry and Stephen Davis

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4 DIFFERENTIATION

1. Find $\frac{dy}{dx}$ if $y = \frac{2}{x^3}$.

   Solution:

$$\begin{aligned} y' &= 2\frac{d}{dx}x^{-3} \\ &= -6x^{-4} \\ &= \frac{-6}{x^4} \end{aligned}$$

2. Find $\frac{dy}{dx}$ if $y = \frac{4}{x^2}$.

   Solution:

$$\begin{aligned} y &= 4x^{-2} \\ \Longrightarrow \quad y' &= 4.(-2).x^{-3} \\ &= -\frac{8}{x^3} \end{aligned}$$

3. Find $\frac{dy}{dx}$ if $y = x - \frac{1}{x^2}$.

   Solution:

$$\begin{aligned} y &= x - x^{-2} \\ \Longrightarrow \quad y' &= 1 - 1.(-2)x^{-3} \\ &= 1 + \frac{2}{x^3} \end{aligned}$$

4. If $f(x) = 1 - \frac{1}{x^2}$ then $f'(x)$ is

   Solution:

$$\begin{aligned} f(x) &= 1 - x^{-2} \\ \Longrightarrow \quad f'(x) &= 0 - 1.(-2).x^{-3} \\ &= \frac{2}{x^3} \end{aligned}$$

5. If $f(x) = \frac{1}{2x}$ then $f'(x)$ is

Solution:

$$\begin{aligned} f(x) &= \frac{1}{2}x^{-1} \\ \Longrightarrow \quad f'(x) &= \frac{-1}{2}x^{-2} \\ &= \frac{-1}{2x^2} \end{aligned}$$

6. If $f(x) = \frac{1}{3}x^{-3}$ then $f'(x)$ is

Solution:

$$\begin{aligned} f(x) &= \frac{1}{3}x^{-3} \\ \Longrightarrow \quad f'(x) &= -3.\frac{1}{3}x^{-4} \\ &= -x^{-4} \end{aligned}$$

7. Find $\frac{dy}{dx}$ if $y = \sin 2x$.

Solution:
Since

$$\begin{aligned} \frac{d}{dx}\sin x &= \cos x \\ \Longrightarrow \quad y' &= \frac{d2x}{dx}\cos 2x \\ &= 2\cos 2x \end{aligned}$$

8. Find $\frac{dy}{dx}$ if $y = \frac{1}{3}\cos 3x$.

Solution:
Since

$$\begin{aligned} \frac{d}{dx}\cos x &= -\sin x \\ \Longrightarrow \quad y' &= -\frac{1}{3}\frac{d3x}{dx}\sin 3x \\ &= -\sin 3x \end{aligned}$$

9. Find dy/dx if $y = \sin(3x^2)$.

Solution:
Since

$$\begin{aligned} \frac{d}{dx}\sin x &= \cos x \\ \text{and} \quad \frac{d}{dx}x^2 &= 2x \\ \Longrightarrow \quad y' &= 3.2x.\cos(3x^2) \\ &= 6x\cos(3x^2) \end{aligned}$$

10. Find y' if $y = 2\cos(x^2)$.

Solution:
Since

$$\begin{aligned} \frac{d}{dx}\cos x &= -\sin x \\ \text{and} \quad \frac{d}{dx}x^2 &= 2x \\ \Longrightarrow \quad y' &= -2.2x.\sin(x^2) \\ &= -4x\sin(x^2) \end{aligned}$$

11. Find $f'(x)$ if $f(x) = \dfrac{1}{(1-3x)^2}$.

Solution:

$$\begin{aligned} f(x) &= (1-3x)^{-2} \\ \Longrightarrow \quad f'(x) &= (-2)(-3)(1-3x)^{-3} \\ &= \frac{6}{(1-3x)^3} \end{aligned}$$

12. Find $\dfrac{dy}{dx}$ if $y = x^2e^x$.

Solution:
Since $\frac{d}{dx}x^2 = 2x$ and $\frac{de^x}{dx} = e^x$ then

$$\begin{aligned} y' &= \frac{dx^2}{dx}e^x + x^2\frac{de^x}{dx} \\ &= 2xe^x + x^2e^x \\ &= xe^x(2+x) \end{aligned}$$

13. Find $f'(x)$ if $f(x) = \dfrac{x}{x^2-3}$.

Solution:

$$\begin{aligned} f(x) &= \frac{x}{x^2-3} \\ \implies f'(x) &= \frac{\frac{dx}{dx}(x^2-3) - x\frac{d}{dx}(x^2-3)}{(x^2-3)^2} \\ &= \frac{(x^2-3) - x.2x}{(x^2-3)^2} \\ &= \frac{-x^2-3}{(x^2-3)^2} \end{aligned}$$

14. Find $x'(t)$ if $x(t) = e^{-t}\sin(2t)$.

Solution:

$$\begin{aligned} x(t) &= e^{-t}\sin(2t) \\ \implies x'(x) &= \sin 2t\frac{d}{dt}[e^{-t}] + e^{-t}\frac{d}{dt}[\sin 2t] \\ &= \sin 2t(-e^{-t}) + e^{-t}(2\cos 2t) \end{aligned}$$

15. Find $\frac{dy}{dt}$ if $y(t) = e^t(\cos t + \sin t)$.

Solution:

$$\begin{aligned} \implies y'(t) &= (\cos t + \sin t)\frac{d}{dt}[e^t] \\ &\quad + e^t\frac{d}{dt}[\cos t + \sin t] \\ &= e^t(\cos t + \sin t - \sin t + \cos t) \\ &= 2e^t\cos t \end{aligned}$$

16. Find $\frac{dy}{dx}$ if $y = \sin^3(2x)$.

Solution:

$$\begin{aligned} y &= \sin^3(2x) \\ \implies y' &= 3\sin^2(2x)\frac{d}{dx}[\sin 2x] \\ &= 3\sin^2(2x)\frac{d2x}{dx}\cos 2x \\ &= 3\sin^2(2x)2\cos 2x \\ &= 6\sin^2(2x)\cos 2x \end{aligned}$$

17. Find $\frac{dy}{dx}$ if $y = \ln(\ln x)$.

Solution:
since $\frac{d}{dx}\ln x = \frac{1}{x}$ then

$$\begin{aligned} y' &= \frac{1}{\ln x}\frac{d}{dx}[\ln x] \\ &= \frac{1}{\ln x}\,\frac{1}{x} \\ &= \frac{1}{x\ln x} \end{aligned}$$

18. Find $\frac{dy}{dx}$ if $y = \sin(x + x^3)$.

Solution:

$$\begin{aligned} y &= \sin(x + x^3) \\ \Longrightarrow \quad y' &= \cos(x + x^3)\frac{d}{dx}[x + x^3] \\ &= \cos(x + x^3)(1 + 3x^2) \end{aligned}$$

19. Find dy/dx if $y = x\cos x$.

Solution:
Since $\frac{d}{dx}\cos x = -\sin x$ then by the product rule

$$\begin{aligned} y' &= \cos x\frac{d}{dx}x + x\frac{d}{dx}\cos x \\ &= \cos x - x\sin x \end{aligned}$$

20. Find $\frac{dy}{dx}$ if $y = x\sin x$.

Solution:

$$\begin{aligned} y &= x\sin x \\ \Longrightarrow \quad y' &= \frac{d}{dx}[x]\sin x + x\frac{d}{dx}[\sin x] \\ &= \sin x + x\cos x \end{aligned}$$

21. Find $\frac{dy}{dx}$ if $y = x^2\sin x$.

Solution:

$$\begin{aligned} y &= x^2 \sin x \\ \implies y' &= \frac{d}{dx}[x^2] \sin x + x^2 \frac{d}{dx}[\sin x] \\ &= 2x \sin x + x^2 \cos x \end{aligned}$$

22. Find $\dfrac{dy}{dx}$ if $y = x(1-x)^4$.

Solution:

$$\begin{aligned} y &= x(1-x)^4 \\ \implies y' &= \frac{d}{dx}[x](1-x)^4 + x\frac{d}{dx}[(1-x)^4] \\ &= (1-x)^4 + x(4(1-x)^3)\frac{d}{dx}[1-x] \\ &= (1-x)^4 + 4x(1-x)^3(-1) \\ &= (1-x)^4 - 4x(1-x)^3 \end{aligned}$$

23. Find $\dfrac{dy}{dx}$ if $y = x(x+2)^6$.

Solution:

$$\begin{aligned} y &= x(x+2)^6 \\ \implies y' &= \frac{d}{dx}[x](x+2)^6 \\ &\quad +x\frac{d}{dx}[(x+2)^6] \\ &= (x+2)^6 + x6(x+2)^5 \end{aligned}$$

24. Find $\dfrac{dy}{dx}$ if $y = xe^x$.

Solution:

$$\begin{aligned} y &= xe^x \\ \implies y' &= \frac{d}{dx}[x]e^x + x\frac{d}{dx}[e^x] \\ &= e^x + xe^x \end{aligned}$$

25. Find $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right)$.

Solution:

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{\frac{d}{dx}[\sin x]x - \sin x\frac{d}{dx}[x]}{x^2} = \frac{\cos(x)x - \sin x}{x^2}$$

26. Find $\frac{d}{dx}\left(\cos^2(3x)\right)$.

Solution:

$$\begin{aligned} ddx\left(\cos^2(3x)\right) &= 2\cos(3x)\frac{d}{dx}[\cos 3x] \\ &= 2\cos(3x)(-\sin 3x)\frac{d}{dx}[3x] \\ &= 2\cos(3x)(-\sin 3x)3 \\ &= -6\cos 3x \sin 3x \end{aligned}$$