

# MATH111 – Spring 2007

## Tutorial Sheet – Week 6

This tutorial sheet principally covers chapter 6 of the notes. Note that the first part of this tutorial sheet is to be detached from the tutorial sheet and handed in at the end of the tutorial.

### Part One

Name \_\_\_\_\_

1 Draw a concept map for chapters 1–6.

**2** What do you consider to be the four most important ideas discussed in chapters 1–6? Write a paragraph describing each idea and explaining why you consider it important. Provide appropriate mathematical examples.

## Part Two (Chapter 6)

### Revision of Key Ideas

1. The discrete logistic difference equation is

$$x_{n+1} = rx_n(1 - x_n).$$

Write down the logistic difference equation with proportional harvesting and with fixed harvesting.

2. Show that the unscaled population model with fixed harvesting

$$N_{n+1} = N_n \left( r - \frac{N_n}{K} \right) - H.$$

can be written in the form

$$x_{n+1} = rx_n(1 - x_n) - h,$$

by introducing a new variable  $x_n = \frac{N_n}{rK}$ . How is the parameter  $h$  defined?

3. Determine the maximum sustainable harvest in the discrete logistic equation with fixed harvesting,

$$x_{n+1} = rx_n(1 - x_n) - h.$$

4. Show that the unscaled population model with proportional harvesting

$$N_{n+1} = N_n \left( r - \frac{N_n}{K} \right) - pN_n.$$

can be written in the form

$$x_{n+1} = rx_n(1 - x_n) - px_n,$$

by introducing a new variable  $x_n = \frac{N_n}{rK}$ .

5. Determine the maximum sustainable yield in the discrete logistic equation with proportional harvesting,

$$x_{n+1} = rx_n(1 - x_n) - px_n.$$

6. Explain what the term  $-px_n$  'means' in the proportional harvesting model.

7. Derive the equilibrium yield equation for the discrete logistic equation with proportional harvesting,

$$x_{n+1} = rx_n(1 - x_n) - px_n.$$

8. Comment on the relative advantages and disadvantages of fixed and proportional harvesting,

9. Describe two issues that should be included in more detailed models for harvesting that were not considered in this chapter.

### Exercises

1. **ADFA Quiz 2002.**

Consider the logistic equation with fixed harvesting

$$x_{n+1} = rx_n(1 - x_n) - h.$$

Suppose that  $r = 2$  and  $h = 0.05$ .

- (a) Show that the fixed points of this model are  $x_2^* = 0.44365$  and  $x_1^* = 0.05635$  (both correct to five decimal places).  
 (b) Calculate the stability of the two fixed points.

- (c) (i) Sketch the functions  $y = x$  and  $y = 2x(1 - x) - 0.05$ , indicating the location of the fixed points.  
(ii) Using your diagram and your answers to the previous parts of this question explain how the long-term population density depends upon the initial value  $x_0$  ( $0 \leq x_0 \leq 1$ ).  
(iii) A company harvests deer from a wilderness area. Due to a drought the population density changes from 0.44365 to 0.02. Should the company change its harvesting strategy? Give reasons.

2. **ADFA Quiz 2001.**

Consider the logistic equation with proportional harvesting

$$u_{n+1} = ru_n(1 - u_n) - pu_n.$$

- (a) Show that the fixed points of this model are  $u^* = 0$  and  $u^* = \frac{r-1-p}{r}$ .  
(b) Show that the trivial fixed point is stable if

$$-1 < r - p < 1,$$

whilst the nontrivial fixed point is stable if

$$-1 < 2 - r + p < 1.$$

- (c) Suppose that the species  $u$  is a type of fish in the sea. When  $r = 1.6$  a government proposes to allow its fishing fleet to harvest 80% of the population each year. Comment on this strategy.