1 Differential Equations: Harvesting (Chapter13)

1. The logistic differential equation is
$$x' = rx\left((1-\frac{x}{K}\right).$$
Write down the logistic differential equation with constant yield harvesting and with constant effort harvesting.

2. Under what conditions does 'constant yield harvesting' commonly occur?

3. Determine the maximum sustainable rate of harvesting in the logistic difference equation with constant yield harvesting,
$$x_{n+1} = rx_n\left((1-x_n\right) - H.$$

4. Determine the maximum sustainable rate of harvesting in the logistic differential equation with constant yield harvesting,
$$x' = rx\left((1-\frac{x}{K}\right) - H.$$

5. Suppose that the function $\mathcal{G}$ depends upon the state variable x. A *limit-point bifurcation* occurs when
$$\mathcal{G}(x) = \mathcal{G}_x(x) = 0.$$
   (a) Consider the logistic difference equation with constant yield harvesting,
   $$x' = rx\left((1-\frac{x}{K}\right) - H.$$
   Write the steady-state equation in the form
   $$\mathcal{G}(x^*) = 0$$
   and calculate
   $$\mathcal{G}_x(x^*).$$
   Hence determine the values of the fixed-point x^* and the harvesting parameter H at which a limit-point bifurcation occurs. Comment on your finding.

   (b) Consider the logistic difference equation with constant yield harvesting,
   $$x_{n+1} = rx_n\left((1-x_n\right) - H.$$
   Write the fixed point equation in the form
   $$\mathcal{G}(x^*) = 0$$
   and calculate
   $$\mathcal{G}_x(x^*).$$
   Hence determine the values of the fixed-point x^* and the harvesting parameter H at which a limit-point bifurcation occurs. Comment on your finding.

   (c) Consider the modified logistic equation with constant yield harvesting
   $$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left[1-\left(\frac{x}{K}\right)^{\theta}\right] - H.$$
   By finding the limit point bifurcation, or otherwise, show that the critical rate of harvesting is given by
   $$H_{\mathrm{cr}} = \frac{rK}{(1+\theta)^{1/\theta}}\left[1-\frac{K^{\theta}}{1+\theta}\cdot\frac{1}{K^{\theta}}\right] = \frac{rK\theta}{(1+\theta)^{1+1/\theta}}.$$

6. Explain what the term $-Ex$ 'means' in the constant effort harvesting model.

7. For real fisheries is it always reasonable to assume that the number of fish caught per unit tim proportional to the effort expended in fishing?

8. Derive the equilibrium yield equation for the differential logistic equation with constant effort harvest
$$x' = rx\left((1-\frac{x}{K}\right) - Ex,$$
and determine the maximum sustainable yield.

9. Consider the modified logistic equation with constant effort harvesting
$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left[1-\left(\frac{x}{K}\right)^{\theta}\right] - Ex.$$
   (a) Find the steady-state solutions of this equation and determine their stability.

   (b) Derive the equilibrium yield equation for this model.

   (c) Hence obtain the value of the effort E which maximises the rate of harvesting. What is the maxim sustainable yield?

10. Comment on the relative advantages and disadvantages of constant yield harvesting and constant ef harvesting.

11. Sketch the steady-state diagram for the logistic differential equation with constant yield harvesting. I cuss the implications of your figure.

12. Sketch the steady-state diagram for the logistic differential equation with constant effort harvesti Discuss the implications of your figure.