

THE UNIVERSITY OF WOLLONGONG
SCHOOL OF MATHEMATICS AND APPLIED STATISTICS

MATH111: Applied Mathematical Modelling 1

Skills Test: Difference equations

Time allowed : 40 minutes

Number of questions: 15 multiple choice

Attempt all questions.

***CALCULATORS
ARE PERMITTED.***

Please Note: Examination Paper is printed on both sides.

1. Consider the difference equation $a_{n+1} = -3a_n + 5$ with the initial condition $a_0 = 1$. What is a_3 ?
 - (a) $a_3 = 8$.
 - (b) $a_3 = -19$.
 - (c) $a_3 = -1$.
 - (d) $a_3 = 2$.

2. Which difference equation represents the sequence $1, 3, 6, 10, 15, 21, \dots$?
 - (a) $a_n = a_{n-1} + 2$
 - (b) $a_n = a_{n-1} + 3$
 - (c) $a_n = a_{n-1} + n - 1$
 - (d) None of the above

3. You open a bank account with \$500. You add \$25 to your account each month, and the bank pays you 2.4% interest compounded month. Which difference equation describes how your account balance changes from month to month?
 - (a) $a_{n+1} = 0.2a_n + 25 + 500$ with $a_0 = 0$
 - (b) $a_{n+1} = 0.002a_n + 25$ with $a_0 = 500$
 - (c) $a_{n+1} = 1.002a_n + 500$ with $a_0 = 25$
 - (d) $a_{n+1} = 1.002a_n + 25$ with $a_0 = 500$
 - (e) None of the above

4. Suppose that $d_{n+1} = 0.9d_n + 2$. What is the steady-state value?
 - (a) 9
 - (b) 2
 - (c) 20
 - (d) $20/9$
 - (e) None of the above

5. The following difference equation describes the value of a car, where n is in years.

$$V_{n+1} = 0.86V_n.$$

Which of the following is a true statement?

- (a) The value of the car increases 86% each year.
 - (b) The value of the car decreases 86% each year.
 - (c) The value of the car decreases by 14% each year.
 - (d) The value of the car will eventually level out at \$8600.
 - (e) None of the above.
6. The difference equation $a_{n+1} = 1.08a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?
- (a) $a_{10} > 5000$
 - (b) $a_{40} < 5000$
 - (c) It is possible that $a_{30} = 5000$
 - (d) More than one of statements (a–c) could be true
 - (e) None of statements (a–c) has to be true
7. We want to test the solution $a_n = 6n + C$ in the difference equation $a_{n+1} = a_n + 6$. What equation results from substituting the solution into the difference equation?
- (a) $6n + 1 + C = 6n + C + 6$
 - (b) $6(n + 1) + C = 6n + C + 6$
 - (c) $6(n + 1) + C(n + 1) = 6n + Cn + 6$
 - (d) $a_n + 6 = 6(a_n + 6) + C$
 - (e) None of the above
8. $a_n = 3^n \cdot 5 + 2$ is a solution to $a_{n+1} = 3a_n - 4$ with $a_0 = 2$.
- (a) True
 - (b) False

9. The equation $a_{n+1} = na_n + a_n a_{n-1} + n^3$ is nonlinear. Which terms makes it nonlinear?
- (a) na_n
 - (b) $a_n a_{n-1}$
 - (c) n^3
 - (d) All of the above
10. The solution to $a_{n+1} = 5.2a_n$ with $a_0 = 6$ is
- (a) $a_n = 6^n (5.2)$
 - (b) $a_n = 6^n C$
 - (c) $a_n = 5.2^n (6)$
 - (d) $a_n = 5.2n + 6$
 - (e) $a_n = 5.2^n (3)$
11. Which best describes the long-term behaviour of the solution $a_n = (-2)^n (12)$?
- (a) As n gets large, the values of a_n grow without bound.
 - (b) As n gets large, the values of a_n decrease without bound.
 - (c) As n gets large, the values of a_n oscillate, getting farther and farther away from zero.
 - (d) As n gets large, the values of a_n oscillate, getting closer and closer to zero.
 - (e) None of the above.
12. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = 5$ is
- (a) $a_n = 4^n (-1) - 1$
 - (b) $a_n = 4^n (5) - 1$
 - (c) $a_n = 4^n (4) - 1$
 - (d) $a_n = 4^n (6) - 1$
 - (e) None of the above

13. Discuss the long-term behaviour of the solution $a_n = 0.7^n C + 2$ with $a_0 = 1$.
- (a) This solution will increase, converging to the steady-state value of 2.
 - (b) This solution will decrease forever.
 - (c) This solution will increase forever.
 - (d) None of the above.
14. The solution to a difference equation is $a_n = \frac{17}{2} + \frac{5}{2}2^n$. What was the initial condition, a_0 ?
- (a) $\frac{17}{2}$
 - (b) $\frac{5}{2}$
 - (c) 2
 - (d) 11
15. Upon graduation from university you land a job with a starting salary of \$35,000. You are told that as long as your performance is up to par, you can expect a 3% raise each year. Additionally, at the end of your first year you will receive a bonus of \$100, and at the end of each year after that you will receive a bonus equal to \$100 times the number of years you have completed. On the first day of your job, you open a bank account with \$200. This account will earn interest at a rate of 5% compounded annually, and you decide that each year you will deposit your bonus into the account. If a_{n-1} represents the amount of money after $n - 1$ years, which of the following difference equations models your account balances?
- (a) $a_n = 1.03a_{n-1} + 100 + 35,000$
 - (b) $a_n = 1.05a_{n-1} + 100$
 - (c) $a_n = 1.05a_{n-1} + 100n$
 - (d) $a_n = 1.05a_{n-1} + 1.05(100)(n - 1)$
 - (e) $a_n = 1.03a_{n-1} + 1.05(100) + 200$
 - (f) None of the above


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Spring Session 2007

2007 Skills Test A

Student Name: _____ Student Number: _____

Answer Sheet

Completely fill in the appropriate box for each question: 

1. A B C D E F

2. A B C D E F

3. A B C D E F

4. A B C D E F

5. A B C D E F

6. A B C D E F

7. A B C D E F

8. A B C D E F

9. A B C D E F

10. A B C D E F

11. A B C D E F

12. A B C D E F

13. A B C D E F

14. A B C D E F

15. A B C D E F

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2007 Skills Test A

Student Name: Correct Answers Student Number: _____

Answer Sheet

Completely fill in the appropriate box for each question:

1. A

2. D

3. D

4. C

5. C

6. A

7. B

8. B

9. B

10. C

11. C

12. D

13. A

14. D

15. C