School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 4

Spring 2007

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 5

The assignment that you had in *must* include a cover page. On the cover-page you should briefly answer the following questions

(a) What topic did you believe was the most important in the assignment?

(b) Why do you believe that is the most important topic?

(c) What problems did you have with the assignment, if any?

You should answer each question with a complete sentence.

If you fail to provide a cover-page your assignment will automatically be marked 'unsatisfactory'.

You may choose to answer one of the questions on this assignment sheet by working in a group of upto four individuals. If you choose this option then at the end of your group answer you must list the members of your group.

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
Assignment Week 4
Spring 2007 Submission Receipt

Student Name: _____ *Student Number*: _____

Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

1. At the beginning of each year you invest $q\%$ of you yearly salary into a superannuation scheme. Your employer tops up your investment by adding $r\%$ of your salary. Your money draws interest of $p\%$ compounded yearly.

   (a) Write down a **word** equation that defines you investment after n payments.

   (b) Suppose that your salary starts at a base level a and that at the start of every year it increases by a constant amount b.

   Write down, formally, the difference equation that describes your investment after n payments have been made. Define **all** variables and explain your terms.

2. A debt of \$50,000 with interest at 11% compounded every two months is paid back by equal payments every two months over fifteen years.

   (a) What is the regular payment, rounded up to the nearest cent?

   (b) How much profit does the lender make on the loan?

3. "If the Indians hand't spent the \$24! In 1626 Peter Minuit, the first governor of New Netherland, purchased Manhattan Island from the Indians for about \$24." [1]

   Assume that the Indians had invested their \$24 in 1626 and that interest is compounded annually at a uniform rate of 7%. (For the 280 years to 1906, this percentage is about average for the stockmarket when it is not crashing [1]).

   [1] R.I. Moritz. Memorabilia Mathematica: the Philomath's Quotation Book. Mathematical Association of America. (1993)

   (a) What would the value of the Indians investment be in 1906?

   (b) What would the value be today? (Make the same assumption regarding the returns over the investment period).

4. "Properties that were out of reach with a 30-year mortgage can suddenly become affordable when the repayments are spread over an extra 10 years". The Sydney Morning Herald (June 21, 2006)

   (a) A loan of \$250,000 is taken to purchase a house. Suppose that interest is compounded monthly at 7%.

      (i) What is the monthly repayment if the loan is paid back over a 30-year mortgage?

      (ii) What is the monthly repayment if the loan is paid back over a 40-year mortgage?

      (iii) How much more does it cost to repay the loan over 40 years?

   (b) Suppose that you have a mortage at which interest is compounded monthly at 7% and that you can afford to pay \$1700 a month.

      (a) If you wish to pay the loan back over 30 years how much money can you borrow from the bank?

      (b) If you wish to pay the loan back over 40 years how much money can you borrow from the bank?

      (c) How much more money can you borrow from the bank if you spread your repayments over forty years?

      (d) By taking a 40-year mortgage how much more money do you end up paying the bank?

      (e) Do you agree with quotation from The Sydney Morning Herald?

5. In the mid-session test and/or the final exam you may be asked a question about Maple.

   This question involves finding the wavelength, L, of a tsunami for a given water depth [2]. The governing equation is

$$f(L) = L - \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right) = 0, \tag{1}$$

   where g is gravity, $T = 2880$ s is the period of the tsunami, and $d \approx 4000$ m is the water depth typical for the Indian Ocean [2].

(a) Plot the function $f(L)$ and graphically estimate the zero. Make sure that your plot contains appropriate axis labels.

Note. In plotting the function you will need to decide upon a suitable horizontal range.

(b) Use `fsolve` to find the zero.

Note. You may need to give maple some 'help' to find the zero. Look at the help page for the `fsolve` command.

(c) (i) Explain the approximation $\tanh x \approx x$.

(ii) Hence, or otherwise, obtain the approximation

$$f(L) \approx L - \frac{gT^2}{L}.$$

(iii) Solve this equation for L_1.

(iv) Compare the approximate answer L_1 against the full numerical solution. What is the error?

Note. Do not use maple for any part of this question.

(d) (i) Explain the approximation $\tanh x \approx x - \frac{x^3}{3}$. (Do not use maple for this question)

(ii) Hence, or otherwise, obtain the approximation

$$L^4 - \left(gT^2 d\right) L^2 + 4gT^2 d^3 \pi^2 \approx 0$$

and use the `solve` command to find the zeroes of this function. Which of these zeroes is the correct answer to the physical problem?

(e) If $d = 4000 \pm 50$ what error does one get using the linear approximation L_1?

[2] S.I. Barry & T. Webb. Multi-disciplinary approach to teaching numerical methods to engineers using Matlab. *ANZIAM Journal* **47**, C216–C230, 2006.