School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 6

Spring 2006

Student Name: ______________________________ *Student Number:*________________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 7

1. Consider the logistic equation with fixed harvesting

$$x_{n+1} = rx_n\left(1 - x_n\right) - h, \quad n = 0, 1, 2 \ldots$$

where $1 < r < 4$ and $0 \leq h \leq 1$.

(a) The stable fixed point of the harvesting model (should it exist) is given by

$$x^* = \frac{-\left(1-r\right) + \sqrt{\left(1-r\right)^2 - 4rh}}{2r}$$

Find the fixed point (to four decimal places), and the associated eigenvalue, when

(i) $r = 1.5$ and $h = 0.015$.

(ii) $r = 1.6$ and $h = 0.05625$.

(iii) $r = 2$ and $h = 0.045$.

(b) Gollum Fresh Fish (motto 'fish fresh from the sea, three times a day') has the choice to send its fishing fleet to one of three fisheries. The value for h is regulated by Mordor Moguls. The long-term yearly profit ($\mathcal{P}$) for fishing in a fishery is

$$\mathcal{P} = ax^* - b$$

where a and b are parameters that depend upon the fishery and x^* is the steady fixed point of the harvesting model for the specified values of h and r. The numbers associated with each fishery are

Fishery one: $r = 1.5$, $h = 0.015$, $a = 3$, $b = 0.6$.

Fishery two: $r = 1.6$, $h = 0.05625$, $a = 4$, $b = 0.45$.

Fishery three: $r = 2$, $h = 0.045$, $a = 1$, $b = 0.2$.

Which fishery should Gollum Fresh Fish use and why?

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Tutorial Class: __________ *Date Submitted:* ________________ *Tutor Initials:* ________________

2. Show that the unscaled logistic model with proportional harvesting

$$N_{n+1} = N_n \left(r - \frac{N_n}{K} \right) - pN_n \quad 0 \le p$$

can be written in its standard form

$$x_{n+1} = rx_n \left(1 - x_n\right) - px_n$$

by introducing a new scaling $x_n = \frac{N_n}{rK}$.

3. (a) Find the nonnegative equilibria of a population governed by

$$x_{n+1} = \frac{3x_n^2}{x_n^2 + 2}$$

and check for stability.

(b) Suppose a fraction a is removed from the population in each generation, so that the model becomes

$$x_{n+1} = \frac{3x_n^2}{x_n^2 + 2} - ax_n.$$

For what values of a is there a stable equilibrium only at $x = 0$?

In the mid-session test and/or the final exam you may be asked a question about Maple.

4. The following example is taken from the lecture notes.

The interest on an investment of \$5 000 at the MAS Bank earns 6.5% interest rate compounded monthly. The teller at the bank explains that at the end of every month, the new principal will be worked out using the equation

$$P_{n+1} = \left(1 + \frac{0.065}{12}\right) P_n\,, \qquad P_0 = 5\,000, \quad n = 0, 1, 2, \ldots \tag{1}$$

How much money is in the bank account at the end of the first month?

The answer to this question is that $P_1 = \$5,027.08$. At the Wollongong campus a student rounded their answer down to \$5,027.

Before answering this question you are advised to carefully consider **question 6** from the week two maple worksheet.

(a) A student invests \$5,000 at the MAS Bank at 6.5% compounded monthly. How much money do they have after twenty years? (You do *not* need maple for this part of the question!)

(b) Suppose that at the end of each year MAS Bank rounds your investment down to the nearest dollar. How much money have you lost to the bank after twenty years?

Your answer to (ii) should include all maple code that you used to obtain the answer.