School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 2

Spring 2006

Student Name: ______________________________ *Student Number:* ______________

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 3

1. For the following difference equations: identify the state variable and the 'time' variable; give their order and state whether they are linear, nonlinear, autonomous or non-autonomous.
   (a) $n_{y+2} = n_{y+1}y$
   (b) $y_{n+1} = 2y_{n-1} + sin\,(n)$
2. (i) Show that $x_n = an + b$ is a solution of the difference equation
   $$x_{n+1} - 2x_n + x_{n-1} = 0,$$
   where a and b are constants.
   (ii) Find the solution of the difference equation
   $$n_{x+1} - 2n_x + n_{x-1} = 0, \qquad n\,(x=1) = 7,\, n\,(x=3) = 13.$$
3. Solve the following difference equations to obtain solutions in "closed form".
   (a) $y_{n+1} - \frac{1}{2}y_n = 2, y_0 = c.$
   (b) (i) Evaluate the expression $\sum_{p=1}^{n} 2^{n-p}\cdot 3^p$. Hint Show that
   $$\sum_{p=1}^{n} 2^{n-p}\cdot 3^p = 2^n\left[\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 + \ldots + \left(\frac{3}{2}\right)^n\right].$$
   (ii) Hence solve the difference equation $x_n = 2x_{n-1} + \dfrac{3^n}{3}, \quad x_0 = 0.5.$
4. Consider the problem of modelling the number of of chickens in Mr & Mrs Tweedy's farm. Each week the following activities occur:
   - The number of chickens increases through natural growth by 10%.
   - A fraction, α, of the chickens are killed by foxes.
   - A constant number of chickens are converted into chicken pies.

   (a) Write down a **word** equation that defines this problem.
   (b) Write down, formally, the difference equation that describes the above scenario. Define **all** variables and explain your terms.

School of Mathematics & Applied Statistics **MATH111: Applied Mathematical Modelling 1**
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Student Name: ______________________________ *Student Number*: ______________

Tutorial Class: __________ *Date Submitted:* ______________ *Tutor Initials:* ______________

In the mid-session test and/or the final exam you may be asked a question about Maple.

5. Municipal solid waste (MSW) may contain upto 30-40% of organic materials by mass. These organic wastes should be removed from the MSW before the MSW is delivered to a landfill site. One way to do this is to biologically oxidise the organic fraction.

   The maximum specific oxidation rate (hr^{-1}) in a bioreactor is given by the formula

$$\mu_{\max} = \frac{A' \exp\left[-\frac{E_g}{RT}\right]}{1 + B \exp\left[-\frac{\Delta G_d}{RT}\right]},$$

   where A' (hr^{-1}) and $B\,(-)$ are constants, E_g is the activation energy of the growth process (kJ/mol), and ΔG_d is the Gibbs free energy change upon protein denaturation (kJ/mol).

   In (1) the following parameter values were estimated: $A' = 4.032 \times 10^8\,\text{hr}^{-1}$, $B = 4.776 \times 10^{89}$, $E_g = 56.861\,\text{kJ/mol}$, $\Delta G_d = 537.56\,\text{kJ/mol}$, $R = 8.31431\,\text{kJ/mol/K}$.

   (1) E. Liwarska-Bizukojc, M. Bizukojc and S. Ledakowicz. 2001. Kinetic model for the process of aerobic biodegradation of organic fraction of municipal solid waste. *Bioprocess and Biosystems Engineering*, **24**: 195–202.

   (i) Plot the function $\mu_{\max}$ as a function of temperature over the range $275 \leq T\,(\text{K}) \leq 320$.

   (ii) Find the value of T (to one decimal place) that maximises the value of $\mu_{\max}$.

   Your answer to (i) & (ii) should include all maple code that you used to obtain the answer.