

School of Mathematics & Applied Statistics
**MATH111: Mathematics Applied Mathematical
 Modelling 1**
Assignment Week 10
Spring 2006

Student Name: _____ *Student Number:* _____

FULL WORKING is to be shown for all solutions.
 Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
 This assignment is to be handed in during your tutorial in Week 11

1. Find all the equilibria and determine their stability for the equation

$$\dot{x} = rx \left[1 - \left(\frac{x}{K} \right)^\theta \right], \quad K > 0, \quad 0 < \theta < 1.$$

2. (a) The growth of a tumour inside the human body can be represented by the equation

$$\frac{dT}{dt} = \beta T \left(1 - \frac{T}{K} \right), \quad T(0) = T_0.$$

where T is the size of the tumour, β denotes the growth rate of the tumour and K is the maximum tumour size.

- (i) Sketch the rate of change of tumour growth \dot{T} as a function of T .
 - (ii) Using your sketch describe how the long-term evolution of the differential equation depends upon the choice of the initial condition T_0 .
 - (iii) Suggest biomedical interpretations for the steady-state solutions $T = 0$ and $T = K$.
- (b) The growth of a tumour inside the human body when radiation therapy is used can be represented by the equation

$$\frac{dT}{dt} = \beta T \left(1 - \frac{T}{K} \right) - \mathcal{I}, \quad T(0) = T_0,$$

where the parameter \mathcal{I} is proportional to the intensity of the radiation.

- (i) Find the steady-state solutions of this model and sketch how they vary as a function of the intensity. (Do not calculate stability).
- (ii) Hence, or otherwise, determine a condition for the tumour to be destroyed.
- (iii) Suppose that for a particular patient $K = 500$ and $\beta = 4$ (in appropriate units). Suppose that the value of the irradiance \mathcal{I} can be controlled with an error tolerance of $\pm 1\%$. Suggest a value for \mathcal{I} to destroy the tumour, justifying your answer.

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Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

3. Consider the differential equation

$$\frac{dx}{dt} = -\mu x + \alpha x^3.$$

- Find the steady-state solutions of this equation and determine their stability as a function of the parameter μ . Consider the cases $\alpha > 0$ and $\alpha < 0$ separately.
- Draw a steady-state diagram for this equation, indicating stable and unstable steady-state solutions using solid and dashed lines respectively. Consider the cases $\alpha > 0$ and $\alpha < 0$ separately.
- For what values of the parameter μ does a bifurcation occur? Consider the cases $\alpha > 0$ and $\alpha < 0$ separately.

In the mid-session test and/or the final exam you may be asked a question about Maple.

Your answer should include all maple code that you used to obtain the answer.

4. In order to draw a cobwebbing diagram for the population model

$$x_{n+1} = \frac{5x_n^2}{x_n^2 + 5}, \quad x_0 = A,$$

a figure containing the straight line $y = x$ and the curve $y = \frac{5x^2}{x^2 + 5}$ is required.

- Write maple code to plot the straight line $y = x$ and the curve $y = \frac{5x^2}{x^2 + 5}$ on the same figure over the range $0 \leq x \leq 5$.
- Using your maple generated figure discuss how the long-term dynamics of the population depends upon the value of x_0 .