School of Mathematics & Applied Statistics

MATH111: Mathematics Applied Mathematical Modelling 1

Assignment Week 2

Spring 2004

Student Name: _____ *Student Number:*_____

FULL WORKING is to be shown for all solutions.
Untidy or badly set out work will not be marked and will be recorded as unsatisfactory.
This assignment is to be handed in during your tutorial in Week 3

1. Give the orders of of the following difference equations and state whether they are linear, nonlinear, autonomous or non-autonomous.

   (a) $nx_{n+2} + 3n^2 x_n = x_{n-1} + 2$

   (b) $x_{n-1} + \cosh(x_n) = 2$

2. Consider the difference equation

$$y_k = k y_{k-1}, \quad k = 1, 2, 3 \ldots$$

   with initial condition $y_0 = 1$.

   (a) Calculate y_1, y_2, y_3, y_4 and make a guess at the "closed-form" solution of y_k.

   (b) Verify that your formula satisfies the difference equation and the initial condition.

3. Solve the following difference equations to obtain solutions in "closed form".

   (a) $x_n - 2x_{n-1} = 0$

   (b) $x_n = x_{n-1} + 3$

   (c) $x_n + x_{n-1} = n$

   (Hint: Arithmetic-Geometric Series $\displaystyle\sum_{k=1}^{n} (-1)^{n-k} k = \frac{1}{4}(2n+1) - \frac{1}{4}(-1)^n$)

4. Consider the problem of modelling patient flow in a department of geriatric medicine. Each day the following activities occur:

   - A number of new patients are admitted to the department for acute care.
   - A fraction, α, of the current patients are treated and discharged.
   - A fraction, β, of the current patients, unfortunately, die.
   - A fraction of the current patients, γ, are transferred to another section.

   (a) Write down a **word** equation that defines this problem.

   (b) Write down, formally, the difference equation that describes the above scenario. Define **all** variables and explain your terms.

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Tutorial Class: _____ *Date Submitted:* _____ *Tutor Initials:* _____

5. Imagine this scenario, if you will. Economic rationalism has taken hold of your workplace and it's time to renegotiate your contract. Knowing a thing or two about maths, you make the following proposal. "Boss, I've been far too greedy. But I've come to my senses, after reading *Animal Farm*, and propose a new pay scale. Starting tomorrow, I would like you to pay me two cents..." "*It's a deal*" "...raised to the power of the number of days..." "*Sign here!*" "...the commencement of my new..." "*Next!* "...contract."

   Day one, you are paid 2c (2 raised to the power of one). Day two, 4c (2 squared). Day three, 8c (2^3). Day four, 16c (2^4). Day five, 32c. For week one, you take home 62c.

   (a) How much do you take home in week two?

   (b) How much to you take home in week three?

   (c) How much do you take home in week four?

   Based on an article by Jeremy Chunn that appeared in Mens Style *Summer 2003*