

B Sigma notation

B.1 What is Sigma notation?

The symbol “ \sum ” (Sigma) is a notation used to denote the summation of terms and is defined as

$$\sum_{i=1}^n f(i) = f(1) + f(2) + f(3) + \dots + f(n-1) + f(n)$$

For example

- (i) $\sum_{k=1}^5 k^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$
- (ii) $\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2$
- (iii) $\sum_{p=1}^4 p(p+3) = [1 \cdot (1+3)] + [2 \cdot (2+3)] + [3 \cdot (3+3)] + [4 \cdot (4+3)]$
- (iv) $\sum_{i=3}^5 i^{\frac{1}{2}} = 3^{\frac{1}{2}} + 4^{\frac{1}{2}} + 5^{\frac{1}{2}}$
- (v) $\sum_{n=1}^7 3 = \underline{\hspace{4cm}}$

Some common properties:

- (a) $\sum_{k=1}^n c = cn$ (where c is a constant)
e.g. $\sum_{k=1}^{20} 3 = 3 \times 20 = 60$
- (b) $\sum_{k=1}^n cF(k) = c \sum_{k=1}^n F(k)$
e.g. $\sum_{k=1}^n 4n = 4 \sum_{k=1}^n n$
- (c) $\sum_{k=1}^n [F(k) \pm G(k)] = \sum_{k=1}^n F(k) \pm \sum_{k=1}^n G(k)$
e.g. $\sum_{k=1}^n (3n^2 + 4n) = \sum_{k=1}^n 3n^2 + \sum_{k=1}^n 4n = 3 \sum_{k=1}^n n^2 + 4 \sum_{k=1}^n n$

Here are some **very** useful results which, if needed in quizzes and exams, would be provided by the examiner.

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^n k^4 = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}$$

B.2 Questions

1. Find the following [Barry & Davies]

(a) $\sum_{i=1}^4 i^2$

(b) $\sum_{i=1}^6 (i+1)$

(c) $\sum_{i=1}^4 (i+2)$

(d) $\sum_{i=1}^4 (2i+1)$

2. Use induction to prove the following results

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=1}^n k^4 = \frac{n(n+1)(6n^3 + 9n^2 + n - 1)}{30}$$