Analysis of an immobilised enzyme membrane bioreactor model

M.I. Nelson[1], H.S. Sidhu[2], S. Adesina[3]

(1) School of Mathematics & Applied Statistics, University of Wollongong, AUSTRALIA.

(2) School of Physical, Environmental and Mathematical Science, UNSW at ADFA, Canberra, AUSTRALIA

(3) School of Chemical Sciences and Engineering, The University of New South Wales, Sydney, AUSTRALIA

Talk Outline

- Context — what is the problem?
- The Model.
- What does the mathematics tell us?
- Conclusions
- Future Work.

Context

- Immobilised enzyme reactors (IER) are used in many process industries.

Context

- Immobilised enzyme reactors (IER) are used in many process industries.
- Investigate an enzyme-catalysed reaction, obeying Michaelis-Menten kinetics, occurring in an IER.

Context

- Immobilised enzyme reactors (IER) are used in many process industries.
- Investigate an enzyme-catalysed reaction, obeying Michaelis-Menten kinetics, occurring in an IER.
- Compare reactor-only system to a separator-reactor system.

Context

- Immobilised enzyme reactors (IER) are used in many process industries.
- Investigate an enzyme-catalysed reaction, obeying Michaelis-Menten kinetics, occurring in an IER.
- Compare reactor-only system to a separator-reactor system.
  - reactor-only system: substrate & product flow out of the reactor

Context

- Immobilised enzyme reactors (IER) are used in many process industries.
- Investigate an enzyme-catalysed reaction, obeying Michaelis-Menten kinetics, occurring in an IER.
- Compare reactor-only system to a separator-reactor system.
  - reactor-only system: substrate & product flow out of the reactor
  - separator-reactor: permselective membrane on reactor jacket.

Context

- Immobilised enzyme reactors (IER) are used in many process industries.
- Investigate an enzyme-catalysed reaction, obeying Michaelis-Menten kinetics, occurring in an IER.
- Compare reactor-only system to a separator-reactor system.
  - reactor-only system: substrate & product flow out of the reactor
  - separator-reactor: permselective membrane on reactor jacket.
  - separator-reactor: integrates membrane separation with biological transformation.

Context 2 — Chemistry

Context 2 — Chemistry

$$[E] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E-S] \overset{k_2}{\rightarrow} [E-A] + P. \qquad (1)$$

Context 2 — Chemistry

$$[E] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E-S] \overset{k_2}{\rightarrow} [E-A] + P. \quad (1)$$

- Compare performance of separator-reactor and reactor-only system (two indicators).

Context 2 — Chemistry

$$[E] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E-S] \overset{k_2}{\rightarrow} [E-A] + P. \quad (1)$$

- Compare performance of separator-reactor and reactor-only system (two indicators).
- Dependence upon process parameters? (particularly those associated with the membrane)

The Model (1) — Assumptions

The Model (1) — Assumptions

- The IER is a well-stirred flow reactor.

The Model (1) — Assumptions

- The IER is a well-stirred flow reactor.
- Enzyme species are immobilised

The Model (1) — Assumptions

- The IER is a well-stirred flow reactor.
- Enzyme species are immobilised
- The product leaves the bioreactor either in the reactor outflow or through the membrane.

The Model (1) — Assumptions

- The IER is a well-stirred flow reactor.
- Enzyme species are immobilised
- The product leaves the bioreactor either in the reactor outflow or through the membrane.
- The flow of sweep liquid through the jacket is high enough to prevent a concentration gradient along the length of the jacket membrane.

The Model (1) — Assumptions

- The IER is a well-stirred flow reactor.
- Enzyme species are immobilised
- The product leaves the bioreactor either in the reactor outflow or through the membrane.
- The flow of sweep liquid through the jacket is high enough to prevent a concentration gradient along the length of the jacket membrane.
- Fouling of this membrane is neglected.

The Model (2) — Scaled Equations!

Enzyme associated species

$$\theta_S(t) + \theta(t) = 1$$

$$\frac{\mathrm{d}\theta_S}{\mathrm{d}t} = k_1(1-\theta_S)S - (1+k_2)\theta_S$$

Non-enzyme associated species

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{\tau}(S_0 - S) + \theta_S - k_1(1-\theta_S)S$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{1}{\tau}P + k_2\theta_S - U(P - P_j)$$

$$\frac{\mathrm{d}P_j}{\mathrm{d}t} = -\frac{1}{\tau_j}P_j + UV(P - P_j)$$

The Model (2) — Scaled Equations!

Enzyme associated species

$$\theta_S(t) + \theta(t) = 1$$

$$\frac{\mathrm{d}\theta_S}{\mathrm{d}t} = k_1(1-\theta_S)S - (1+k_2)\theta_S$$

Non-enzyme associated species

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{1}{\tau}(S_0 - S) + \theta_S - k_1(1-\theta_S)S$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{1}{\tau}P + k_2\theta_S - U(P - P_j)$$

$$\frac{\mathrm{d}P_j}{\mathrm{d}t} = -\frac{1}{\tau_j}P_j + UV(P - P_j)$$

The Model (3) – analysis

- There is a unique steady-state solution.
- It is stable.

Analysis (1) – P_{tot}

The total steady-state product concentration leaving the reactor is $P_{\text{tot}} = P_j + P$.

Analysis (1) – $P_{\mathbf{tot}}$

The total steady-state product concentration leaving the reactor is $P_{\text{tot}} = P_j + P$.

- For small residence times (τ) membrane extraction increases P_{tot} upto a maximum of twice the concentration recovered when there is no membrane extraction.

Analysis (1) – $P_{\mathbf{tot}}$

The total steady-state product concentration leaving the reactor is $P_{\text{tot}} = P_j + P$.

- For small residence times (τ) membrane extraction increases P_{tot} upto a maximum of twice the concentration recovered when there is no membrane extraction.
- At high residence times (τ) P_{tot} is *lower* in a separator-reactor system than a reactor-only system.

Analysis (1) – $P_{\mathbf{tot}}$

The total steady-state product concentration leaving the reactor is $P_{\text{tot}} = P_j + P$.

- For small residence times (τ) membrane extraction increases P_{tot} upto a maximum of twice the concentration recovered when there is no membrane extraction.
- At high residence times (τ) P_{tot} is *lower* in a separator-reactor system than a reactor-only system.
- If $\tau < \tau_{\text{cr}}$ then a separator-reactor system is superior.
  If $\tau > \tau_{\text{cr}}$ then a reactor-only system is superior.

Analysis (1) – P_{tot}

The total steady-state product concentration leaving the reactor is $P_{\text{tot}} = P_j + P$.

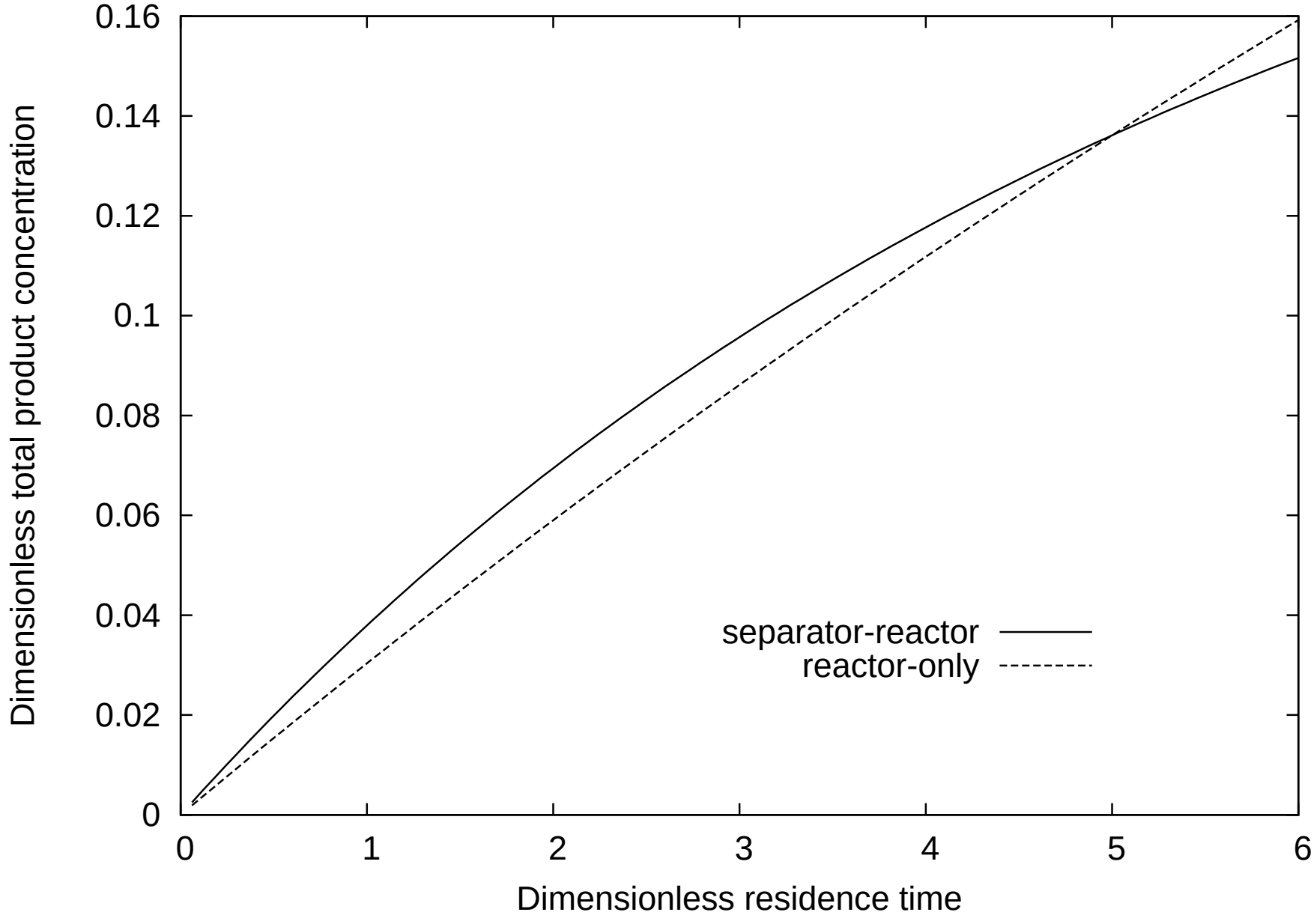
- For small residence times (τ) membrane extraction increases P_{tot} upto a maximum of twice the concentration recovered when there is no membrane extraction.
- At high residence times (τ) P_{tot} is *lower* in a separator-reactor system than a reactor-only system.
- If $\tau < \tau_{\text{cr}}$ then a separator-reactor system is superior.
  If $\tau > \tau_{\text{cr}}$ then a reactor-only system is superior.
- $\tau_{\text{cr}} = \frac{V}{V_j} \cdot \tau_j$

Analysis (2) – $P_{\mathbf{tot}}$

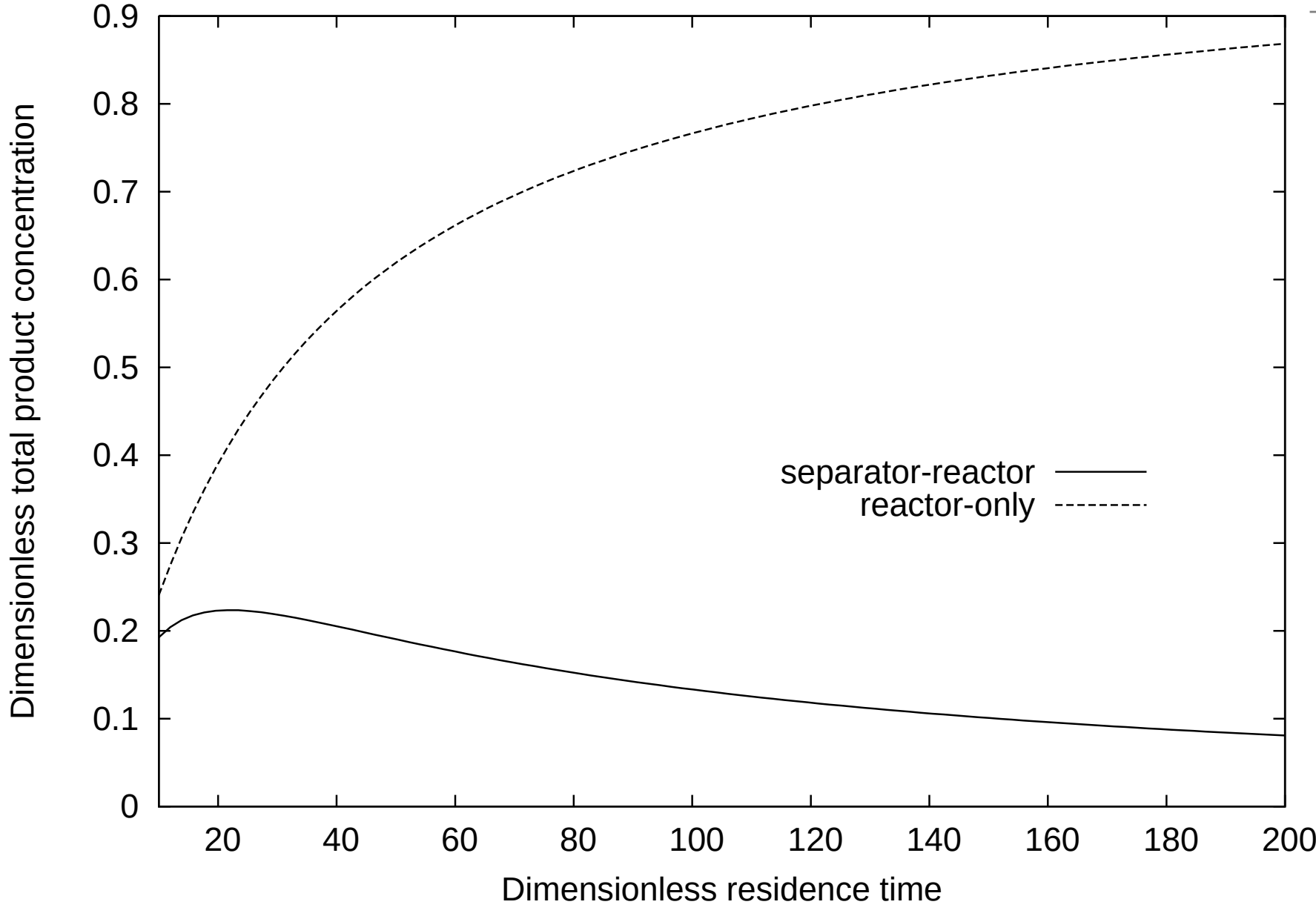
It is also of interest to know the proportion of the total concentration of recovered product that is recovered through the jacket. This is given by

$$\begin{aligned} \mathcal{C}_c &= \frac{P_j}{P_j + P}, \\ &= \frac{UV\tau_j}{1 + 2UV\tau_j}. \end{aligned}$$

Total Concentration (1)



Total Concentration (2)



Analysis (3) — Pr

$$\mathrm{Pr} = \frac{P}{\tau} + \frac{P_j}{\tau_j}$$

Analysis (3) — Pr

$$\mathrm{Pr} = \frac{P}{\tau} + \frac{P_j}{\tau_j}$$

- Pr is an increasing function of V/V_j.

Analysis (3) — Pr

$$\mathrm{Pr} = \frac{P}{\tau} + \frac{P_j}{\tau_j}$$

- Pr is an increasing function of V/V_j.
- Reactor-only system is best if $V/V_j < 1$.

Analysis (3) — Pr

$$\mathrm{Pr} = \frac{P}{\tau} + \frac{P_j}{\tau_j}$$

- Pr is an increasing function of V/V_j.
- Reactor-only system is best if $V/V_j < 1$.
- Separator-reactor system is best if $V/V_j > 1$.

Analysis (3) — Pr

$$\text{Pr} = \frac{P}{\tau} + \frac{P_j}{\tau_j}$$

- Pr is an increasing function of V/V_j.
- Reactor-only system is best if $V/V_j < 1$.
- Separator-reactor system is best if $V/V_j > 1$.
- If $V/V_j > 1$ there is a value of the residence time that maximises the total productivity.

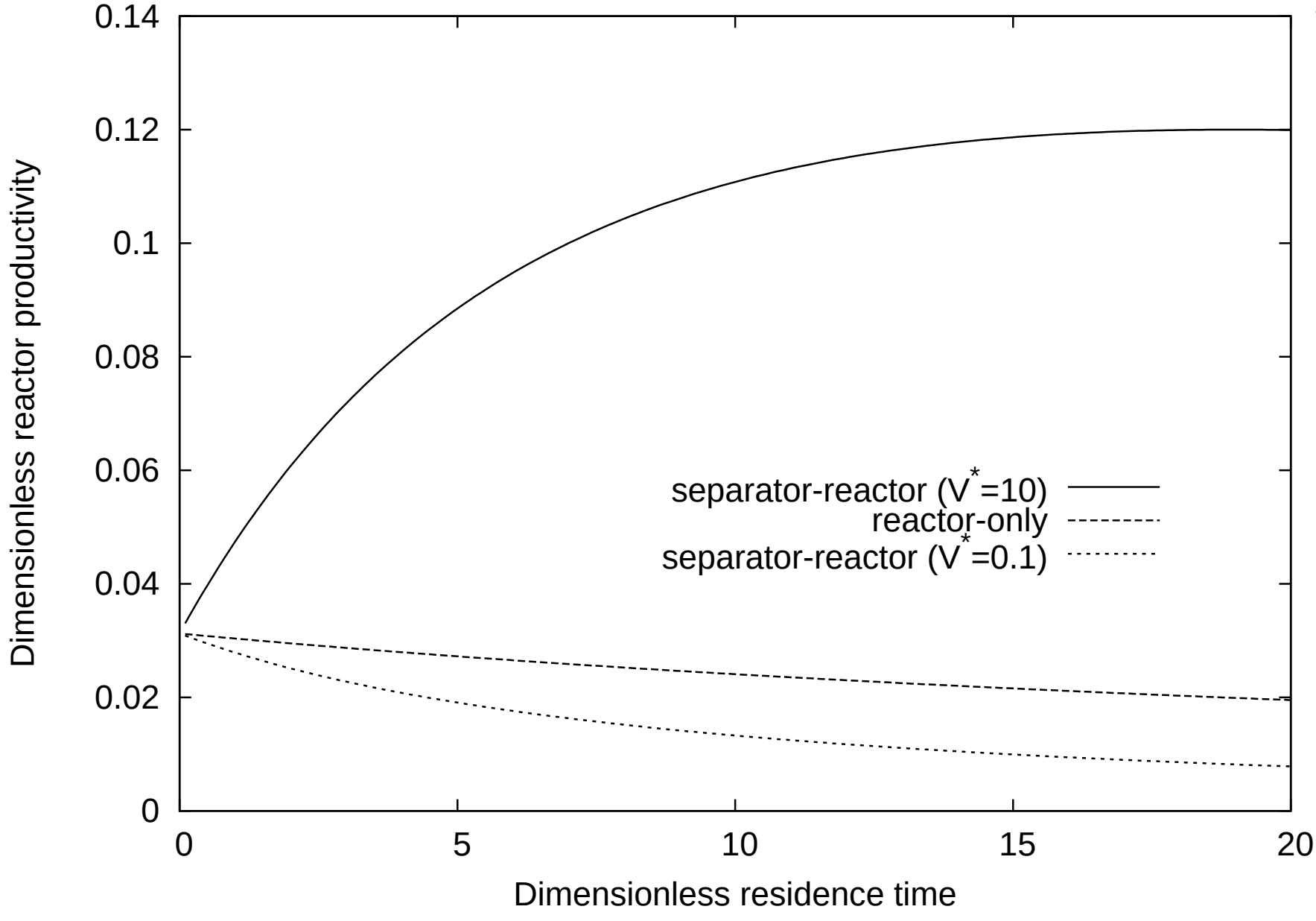
Analysis (4) — Pr

It is also of interest to know the proportion of the total reactor productivity that is recovered through the jacket. This is given by

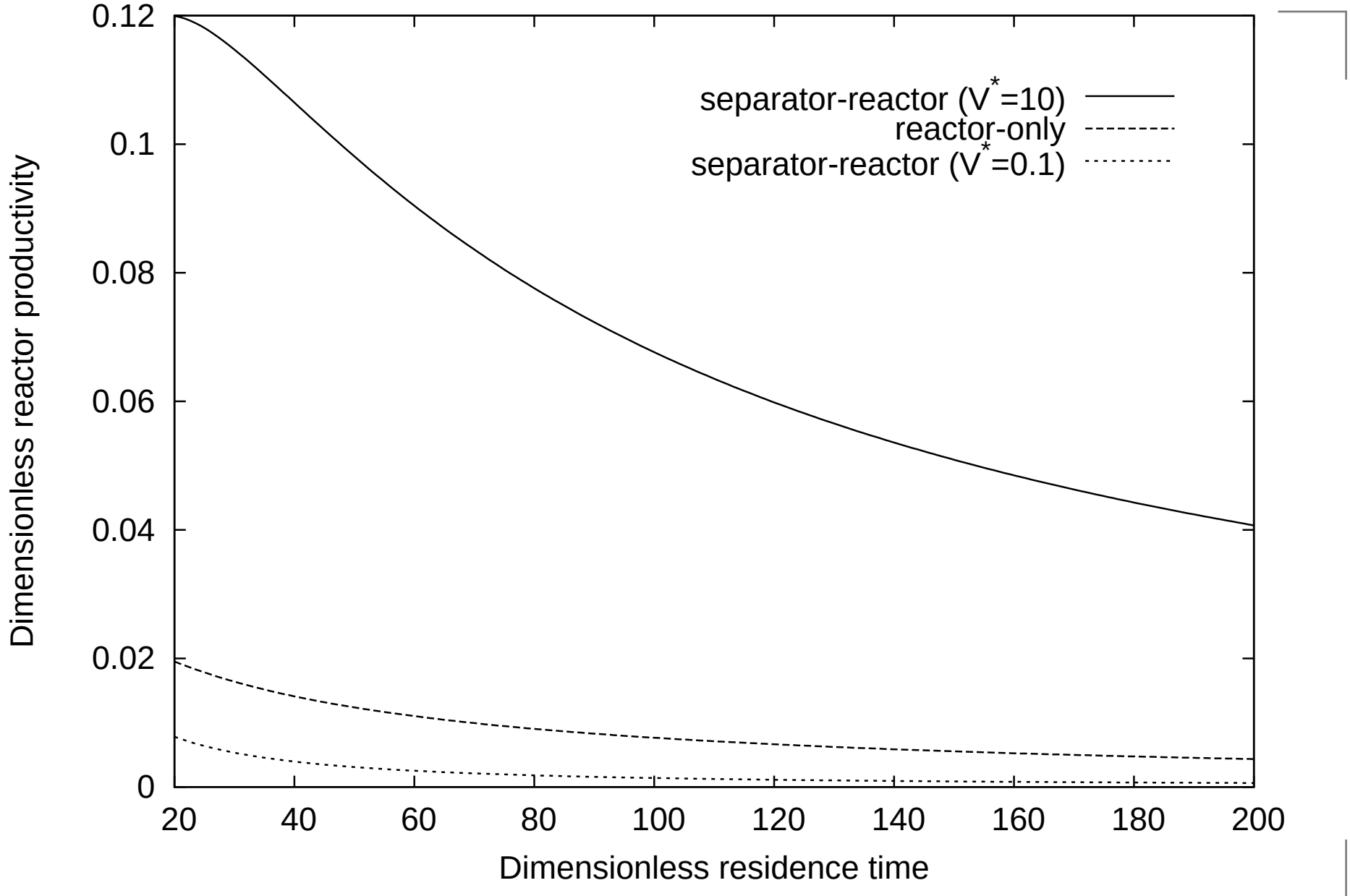
$$\begin{aligned} \mathcal{C}_{\text{pr}} &= \frac{P_j/\tau_j}{P_j/\tau_j + P/\tau}, \\ &= \frac{UV\tau}{1 + UV\left(\tau + \tau_j\right)}. \end{aligned} \tag{1}$$

It immediately follows that the maximum value of $\mathcal{C}_{\text{pr}}$ is 1.0 (100% recovery through the jacket) and occurs in the limit when $\tau \to \infty$.

Total Productivity (1)



Total Productivity (2)



Conclusions – Product Concentration

Investigated the performance of an IER in two configurations: a separator-reactor and reactor-only. The kinetic scheme used was the MM mechanism.

Conclusions – Product Concentration

Investigated the performance of an IER in two configurations: a separator-reactor and reactor-only. The kinetic scheme used was the MM mechanism.

1. At low τ membrane extraction increases the total product concentration (TPC) recovered, whereas at high τ membrane extraction decreases the TPC.

Conclusions – Product Concentration

Investigated the performance of an IER in two configurations: a separator-reactor and reactor-only. The kinetic scheme used was the MM mechanism.

1. At low τ membrane extraction increases the total product concentration (TPC) recovered, whereas at high τ membrane extraction decreases the TPC.
2. Identified τ_{cr} such that if $\tau_{\text{cr}} < \tau$ separator-reactor system is superior and if $\tau_{\text{cr}} > \tau$ reactor-only system is superior.

Conclusions – Product Concentration

Investigated the performance of an IER in two configurations: a separator-reactor and reactor-only. The kinetic scheme used was the MM mechanism.

1. At low τ membrane extraction increases the total product concentration (TPC) recovered, whereas at high τ membrane extraction decreases the TPC.
2. Identified τ_{cr} such that if $\tau_{\text{cr}} < \tau$ separator-reactor system is superior and if $\tau_{\text{cr}} > \tau$ reactor-only system is superior.
3. The ratio of product concentration extracted through the membrane to that in the reactor outflow us $P_j/P = \frac{UV\tau_j}{1+UV\tau_j}$

Conclusions – Productivity

Conclusions – Productivity

1. The key process variable for reactor productivity is $V = V_j/V_r$.

Conclusions – Productivity

1. The key process variable for reactor productivity is $V = V_j/V_r$.
2. If $V > 1$ membrane extraction increases productivity.
   If $V < 1$ membrane extraction decreases productivity.

Future Work

Enzyme activation

Product Inhibition

Future Work

Enzyme activation

$$E + W \underset{k_d}{\overset{k_a}{\rightleftharpoons}} [E - W] + [E - A],$$

$$[E - A] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E - S] \overset{k_2}{\rightarrow} [E - A] + P.$$

Product Inhibition

Future Work

Enzyme activation

$$E + W \underset{k_d}{\overset{k_a}{\rightleftharpoons}} [E - W] + [E - A],$$

$$[E - A] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E - S] \overset{k_2}{\rightarrow} [E - A] + P.$$

Product Inhibition

$$[E] + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E - S] \overset{k_2}{\rightarrow} [E - A] + P,$$

$$E + P \underset{k_d}{\overset{k_a}{\rightleftharpoons}} [E - P].$$