RATIONAL EATING:
CAN IT LEAD TO OVERWEIGHTNESS OR UNDERWEIGHTNESS?*

Amnon Levy, Department of Economics, University of Wollongong


Although a deviation from the physiologically optimal weight increases the probability of dying, the steady state for an expected lifetime-utility maximiser is a state of overweightness. However, even a small initial deviation from this rationally stationary weight is followed by explosive oscillations. These oscillations might lead to severe and chronic underweightness in a late stage of life. In the presence of socio-cultural norms of appearance, the rationally stationary weight of fat people is lower than otherwise and the rationally stationary weight of lean people is greater than otherwise. (JEL I12)

Key words: Eating, weight

1. Introduction

Cycles in food consumption (binges and strict diets in particular) and overweightness and underweightness (obesity and anorexia in particular) can result from psychological, physiological and environmental problems. They may also reflect attempts to conform to social and cultural norms of appearance. This paper shows that in a stylised world where there are no psychological, physiological and environmental problems and where no social and cultural pressures exist the empirically observed overweightness, underweightness and cyclical food consumption can be caused by a rational non-addictive eating.

In an attempt to explain cycles in food consumption Engelbert Dockner and Gustav Feichtinger (1993) have modified Gary Becker and Kevin Murphy’s (1988) rational addiction model by considering two negatively correlated stocks of consumption capital: an addictive stock and weight. The present analysis offers a different approach whereby eating is neither addictive and, consistently with Karen Dynan’s (2000) empirical

* The paper benefited from a feedback given by Joao Faria and David Adelman. An earlier version of this paper was presented in The Nineteenth Economic Theory Workshop hosted by the University Technology Sydney in February 2001. The author is indebted to the discussant of the paper and the participants of this workshop for helpful comments. The author is also indebted to the anonymous referees of this paper for useful comments.
findings with panel household data, nor a formed habit. The possibility of rational cyclical food consumption, as well as overweightness and underweightness, is explained by assuming that utility is derived by consuming food (taken as a homogeneous aggregate), that for any individual there is a *physiologically optimal weight*, that the larger the deviation from the physiologically optimal weight the higher the probability of dying, and that a rational person considers the risk stemming from becoming overweight, or underweight, and plans his, or her, food-consumption trajectory so as to maximise expected utility over her, or his, lifetime, itself affected by food consumption.

A *rationally optimal weight* trajectory is defined as the weight trajectory associated with the food-consumption path which maximises the individual’s expected lifetime utility. The *rationally optimal weight* may not necessarily be equal, or converging, to the *physiologically optimal weight*. A positive difference between the rationally optimal weight and the physiologically optimal weight indicates the individual’s rationally optimal level of overweightness, whereas a negative weight differential reflects his, or her, rationally optimal level of underweightness. Overweightness and underweightness can be considered to be *chronic* if the rationally optimal weight-trajectory does not converge to the physiologically optimal weight and can be diagnosed as *acute* if the rationally optimal weight trajectory diverges from the physiologically optimal weight.

A manageable, stylised, basic, dynamic model of rational, non-addictive, eating model is developed. It is shown that when physiological, psychological, environmental and socio-cultural reasons for divergence from a physiologically optimal weight do not exist, the steady state is a state of overweightness. However, it is asymptotically unstable: even a small initial deviation from this stationary weight is followed by explosive oscillations and binges which might lead in a late stage of life to severe and chronic underweightness. The model is extended to the case where socio-cultural norms of appearance exist. It is demonstrated that in the presence of such norms the stationary weight of fat people is lower than otherwise and the stationary weight of lean people is greater than otherwise.

The paper is structured as follows. A basic dynamic model of rational non-addictive eating is introduced in section 2. The stationary weight advocated by the model is described in section 3. The possibility of convergence to, or divergence from, an initial weight to the stationary rationally optimal weight and cyclical consumption is analysed in
section 4. The effect of socio-culturally preferred weight is analysed in section 5. Comments about possible extensions of the basic model are made in section 6.

2. A model of rational eating

There is a trade-off between satisfaction from eating and risk for life from being overweight or underweight. It is postulated that rational individuals recognise this trade-off and choose their path of food consumption so as to maximise their expected lifetime-utility. The modeling of their expected lifetime-utility and the trade-off between the satisfaction and risk associated with eating employs the following assumptions. The lifetime-utility function is additively separable and the instantaneous utility, $U$, diminishingly increases with food consumption, $c$ (i.e., $U_c > 0$ and $U_{cc} < 0$). The probability of dying, $p$, at time $t$ rises with the quadratic deviation of weight, $W$, from the physiologically optimal weight $W^*$ (i.e., $p_{(W-W^*)^2} > 0$). The rate of time preference is a positive scalar $\rho$. There is an upper-bound, $T$, on life expectancy.

Since life expectancy is random, expected-lifetime-utility-maximising food consumers multiply their accumulated utility from consumption between the starting point of their planning horizon, $0$, to their possible time of death $t$ (i.e., $\int_0^t e^{-\rho \tau} U(c(\tau)) d\tau$) by the probability of dying at time $t$ (i.e., $p((W(t)-W^*)^2)$). The products of $p((W(t)-W^*)^2)$ and $\int_0^t e^{-\rho \tau} U(c(\tau)) d\tau$ associated with any possible life expectancy $0 \leq t \leq T$ are considered by such rational, lifetime planners. The sum of all these products is these planners’ expected lifetime-utility. It is given by the following double-integral expression:

$$J = \int_0^T p((W(t)-W^*)^2) \left\{ \int_0^t e^{-\rho \tau} U(c(\tau)) d\tau \right\} dt. \quad (1)$$

Integrating by parts, the expected lifetime-utility can be equivalently rendered by a mathematically more manageable single-integral expression:
where $Φ$ is equal to 1 minus the cumulative density function associated with $p$ and hence indicates the probability of living beyond $t$. A detailed mathematical explanation is given in Appendix A, and an example including bequest is provided by Kamien and Schwartz (1991, Section 9, pp. 61-62).

It is assumed that $Φ$ is diminishing and concave in $(W - W^*)^2$. This assumption and the concavity of $U$ ensure that an interior solution to the rational-food-consumption problem exists. In this framework, $Φ$ can be viewed as an additional discounting factor. That is, the instantaneous utility from consumption is not only discounted by the individual’s degree of impatience ($ρ$), but also by the risk associated with the individual’s deviation from the physiological optimal weight. The reason for this extra discounting is the individual’s aversion toward uncertainty about life expectancy. There can be other reasons leading to a similar formulation of the expected lifetime-utility. For instance, disutility from being physiologically overweight or underweight.

Weight is gained by consuming food and lost through burning calories as displayed, for simplicity, by the following linear motion equation:

$$\dot{W}(t) = c(t) - \delta(W(t))$$  \hspace{1cm} (3)

where $\delta$ is a positive scalar indicating the marginal (and average) effect of weight on burning calories, and consequently loosing weight, in performing various activities. A rational planner is one who chooses a trajectory of $c$ so as to maximise $J$ subject to this weight motion equation. The Hamiltonian corresponding to this optimal-control problem is

$$H(t) = e^{-ρt}U(c(t))Φ((W(t) - W^*)^2) + \lambda(t)[c(t) - \delta W(t)]$$ \hspace{1cm} (4)

where $\lambda$ is a co-state variable indicating the shadow value of weight in present-value utility units (utiles).

By differentiating the Hamiltonian with respect to $C$ and equating to zero we obtain that along the optimal trajectories of food consumption and weight the shadow value of weight is the negative reflection of the marginal utility from food consumption,
discounted by the individual’s degree of impatience and compounded by her, or his, prospects of continuing living:

\[ e^{-\rho t} U_c \Phi((W - W^*)^2) + \lambda = 0 \]  \hspace{1cm} (5)

or, equivalently,

\[ \lambda = -e^{-\rho t} U_c \Phi((W - W^*)^2) \cdot \]  \hspace{1cm} (5’)

The evolution of the shadow value of weight along the optimally rational trajectories is displayed by the adjoint equation

\[ \dot{\lambda} = -\frac{\partial H}{\partial W} = -e^{-\rho t} U(c) \Phi_w ((W - W^*)^2) + \lambda \delta \]  \hspace{1cm} (6)

where

\[ \Phi_w ((W - W^*)^2) = \frac{\partial \Phi((W - W^*)^2)}{\partial (W - W^*)^2} - 2(W - W^*) \cdot \]  \hspace{1cm} (7)

Dividing both sides of the adjoint equation by \( \lambda \) and recalling the optimality condition (5) we obtain that the rate of change of the shadow value of weight along the optimal trajectories is equal to the difference between the constant marginal effect of weight on burning calories (\( \delta \)) and the product of the survival-utility elasticity ratio and the food consumption-weight ratio:

\[ \frac{\dot{\lambda}}{\lambda} = \delta - \frac{\Phi_w / \Phi}{U_c / U} = \delta - \frac{E_\Phi}{E_U} \frac{c}{W} \]  \hspace{1cm} (8)

where the survival and utility elasticities are defined as follows:

\[ E_\Phi \equiv -\Phi_w \frac{W}{\Phi} \]  \hspace{1cm} (9)

and

\[ E_U \equiv U_c \frac{c}{U} \cdot \]  \hspace{1cm} (10)

The set of necessary conditions for maximum expected lifetime-utility also includes the weight-motion equation (3) and a transversality condition which indicates that weight is worthless when the upper bound on life expectancy is reached:

\[ \lambda(T)W(T) = 0 \cdot \]  \hspace{1cm} (11)
Differentiating the optimality condition (5) with respect to $t$ and substituting the right hand side of the adjoint equation (6) into the resultant singular control equation for $\lambda$ and then the right hand side of equation (5') for $\lambda$, imply that the rationally optimal food-consumption and weight trajectories should satisfy:

$$\frac{U_{cc}}{U_c} \frac{\dot{c}}{c} + \frac{\Phi_W}{\Phi} \dot{W} - \frac{U}{U_c} \frac{\Phi_W}{\Phi} = \rho + \delta. \tag{12}$$

Due to its mathematical complexity, there is no simple interpretation for this no-arbitrage rule. Using the aforementioned elasticity terms and the conventional definition of the degree of absolute risk aversion ($R \equiv -U_{cc}/U_c$) and rearranging terms we obtain that, along the rationally optimal trajectory, the rate of change of food consumption is moderated by the individual’s degree of absolute risk aversion (as regards consumption per se and disregarding survival prospects). This rate of change of food consumption is positive (negative) if the survival-utility elasticity ratio, compounded by the ratio of food-consumption to weight, is larger (smaller) than the sum of the rate of time preference, the rate of depreciation of weight and the elasticity of survival amplified by the rate of change of weight (the latter should obey the motion equation (3)):

$$\frac{\dot{c}}{c} = \frac{1}{R} \left\{ \frac{E_{\Phi}}{E_U} \frac{c}{W} - \left[ (\rho + \delta) + \frac{E_{\dot{W}}}{E_W} \right] \right\}. \tag{13}$$

The implications of the no-arbitrage rule (12) for the rationally optimal stationary levels of food-consumption and weight and the possibility of cycles in food consumption and weight are discussed in the following sections with convenient explicit specifications of the individual’s survival probability and utility functions.
3. The steady state: rational overweightness

By setting of \( \dot{c} \) and \( \dot{W} \) to be equal to zero, equation (12) implies that the stationary levels of the rationally optimal food-consumption \( c_{ss} \) and weight \( W_{ss} \) should satisfy:

\[
- \frac{U(c_{ss})}{U_c(c_{ss})} \frac{\Phi_F(W_{ss})}{\Phi(W_{ss})} = \rho + \delta. \tag{14}
\]

Recalling definitions (9) and (10) we obtain that in a rationally optimal steady state the ratio of food consumption to body weight is equal to sum of the time-preference and weight-loosing rates times the stationary utility-survival elasticity ratio:

\[
\frac{c_{ss}}{W_{ss}} = (\rho + \delta) E_U(c_{ss}) \frac{E_F(c_{ss})}{E_F(W_{ss})}. \tag{15}
\]

The following explicit forms of the survival and utility functions that lead to a closed-form solution to the stationary weight are considered. The probability of living beyond \( t \) is assumed to be given by:

\[
\Phi = \Phi_0 e^{-\mu(W-W^*)^2} \tag{16}
\]

where \( \mu \) is a positive scalar and \( 0 < \Phi_0 < 1 \). In this context, \( \Phi_0 \) is the upper bound of the probability of living beyond \( t \). It indicates the prospects of survival for a person having the physiologically optimal weight. As the actual weight departs (upward or downward) from the physiologically optimal weight, the individual’s prospects of survival diminish gradually and asymptotically at the rate of \( \mu \). The instantaneous satisfaction from eating is assumed to be given by:

\[
U = c^\beta \tag{17}
\]

where \( 0 < \beta < 1 \) is the utility elasticity.

The substitution of equations (16) and (17) and their derivatives into equation (14) implies that the stationary levels of the rationally optimal food-consumption and weight should satisfy:

\[
c_{ss} (W_{ss} - W^*) = \frac{(\rho + \delta) \beta}{2\mu}. \tag{18}
\]

Recalling also that the weight-motion equation (3) implies that in steady state:
equation (18) can be rendered as a second-order polynomial of $W_{ss}$:

$$W_{ss}^2 - W^*W_{ss} - \frac{(\rho + \delta)\beta}{2\delta \mu} = 0.$$  \hspace{1cm} (20)

The only relevant and feasible solution for this polynomial is:

$$W_{ss} = 0.5W^* + 0.5\sqrt{W^*^2 + 2\beta(\rho + \delta)/\delta \mu}.$$  \hspace{1cm} (21)

This solution reveals that the rationally optimal stationary weight is greater than the physiologically optimal weight. It also indicates that the rationally optimal stationary level of overweightness rises with the elasticity of utility ($\beta$) and the rate of time preference ($\rho$). The underlying rationale is that a higher value of $\beta$ is associated with a higher level of satisfaction from eating at any instance, and a higher value of $\rho$ is associated with a declining appreciation of the satisfaction from future food consumption and diminishing concerns of the risk associated with future divergence from the physiologically optimal weight. As can be intuitively expected, equation (21) indicates further that the rationally optimal stationary level of overweightness declines with the calories-burning rate ($\delta$) and the rate of decline ($\mu$) of the probability of living beyond $t$ stemming from an infinitesimal rise in the quadratic deviation from the physiologically optimal weight.

4. Can rational non-addictive eating be cyclical? Would it lead to the rationally optimal stationary level of overweightness?

To answer these questions let us construct the possible trajectories of weight and food-consumption satisfying equations (3) and (12) in the phase-plane diagram. (For a description of this technique with economic examples see Chiang, 1992, Kamien and Schwartz, 1991, and Levy, 1992.) Recalling the explicit forms (16) and (17) of the survival and utility functions, equation (12) implies that the optimal rate of change in food consumption is
\[ \dot{c} = \frac{(\rho + \delta) + \left(\frac{2\mu(\beta - 1)}{\beta}\right)(W - W^*)c - 2\mu\delta W^2 + 2\mu\delta W^*W}{\beta - 1}. \] 

(22)

Hence, the isocline \( \dot{c} = 0 \) is given by:

\[ (\rho + \delta) + \left(\frac{2\mu(\beta - 1)}{\beta}\right)(W - W^*)c - 2\mu\delta W^2 + 2\mu\delta W^*W = 0 \] 

and, totally differentiating equation (23), the slope of this isocline is:

\[ \frac{dc}{dW} \bigg|_{\dot{c}=0} = -\left[\frac{\delta\beta}{1 - \beta} + \frac{c + \frac{\delta\beta}{1 - \beta}}{W - W^*}\right]. \] 

(24)

The slope of the isocline \( \dot{c} = 0 \) is, therefore, negative for \( \tilde{W} > W > W^* \) and positive for \( \tilde{W} < W < W^* \) as displayed in Figure 1, where:

\[ \tilde{W} = W^* - \left[1 + \left(\frac{1 - \beta}{\delta\beta}\right)c\right]. \] 

(25)

The direction of the change in food consumption above and below the isocline \( \dot{c} = 0 \) is obtained by differentiating equation (22) with respect to \( W \) and displayed by the vertical arrows.

By virtue of equation (3), the isocline \( \dot{W} = 0 \) is the locus of all the combinations of \( C \) and \( W \) for which:

\[ c = \delta W. \] 

(26)

It is displayed in Figure 1 by a line whose slope is equal to \( \delta \). Since the derivative of equation (3) with respect to \( c \) is positive, \( \dot{W} \) is positive (negative) above (below) the line as displayed by the horizontal arrows.

*Figure 1 to be inserted here.*

The rationally optimal stationary level of overweightness, which was analysed in the previous section, is now displayed by the intersection point of the two isoclines. The directions of the horizontal and vertical arrows in the phase-plane diagram suggest that this stationary point can be either a spiral or a centre. The steeper the isocline \( \dot{c} = 0 \)
and the flatter the isocline \( \dot{W} = 0 \) (i.e., the smaller \( \delta \)) the larger the fluctuations around the stationary point. That is, the model suggests that binges and strict diets, and more generally cyclical food consumption and weight, may be rational. As demonstrated in the following, the food consumption and weight cycles are explosive and the stationary level of overweightness is unstable.

The linearization of the dynamic system consisting of equations (3) and (22) at the vicinity of steady state implies that the 2x2 state-transition (or Jacobian) matrix \( a \) has the following elements:

\[
\begin{align*}
    a_{11} &= 1 \\
    a_{12} &= -\delta < 0 \\
    a_{21} &= -\left( \frac{\rho + \delta}{1 - \beta} \right) + \left( \frac{2c_{ss} + \delta W_{ss}}{\beta} \right) 2\mu(W_{ss} - \bar{W}) \\
    a_{22} &= \frac{2\mu c_{ss}^2}{\beta} + \frac{4\mu \delta c_{ss}}{1 - \beta} (W_{ss} - 0.5\bar{W}) > 0
\end{align*}
\]

where the sign of \( a_{21} \) is not clear. Note also that the effects of \( \mu, \rho \) and \( \beta \) on \( a_{21} \) are not clear as their direct and indirect (through \( W_{ss} \)) effects on \( a_{21} \) have opposite directions. The eigenvalues of \( a \) are:

\[
\lambda_{1,2} = 0.5 \left( a_{11} + a_{22} \right) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12})}.
\]

Recalling that the directions of the horizontal and vertical arrows in the phase-plane diagram imply that the stationary point is either a spiral or a centre, the discriminant in equation (28) is negative and the eigenvalues are conjugate-complex pair. Since their real part \( a_{11} + a_{22} \) is positive, the stationary point is asymptotically unstable as illustrated in Figure 1 by a diverging spiral. That is, there is no convergence to the stationary level of overweightness but rather explosive oscillations around it.

As illustrated by the trajectory starting at point \( A \) near the steady state, there is also a possibility of a chronic loss of weight (below the physiologically optimal level) in a late stage of life. The greater the individual’s calories-burning rate (\( \delta \)), the steeper the isocline \( \dot{W} = 0 \) and the greater the likelihood of becoming chronically and fatally underweight in old age. In reality, such a chronic decline in food consumption and loss of
weight in a late stage of life might be caused by a high rate of calories burning accompanied by a loss of appetite. However, a loss of appetite is due to factors which are not incorporated into the model. The set of these factors may include uncontrolled deterioration of body systems and mental depression stemming from a diminishing active role in society and family and from a grief associated with a loss of partners and friends in advanced age.

The model’s prediction of fluctuations in individual’s food consumption and weight is consistent with the observed phenomenon of binges followed by strict diets. These fluctuations reflect the conflict between the individual’s myopic inclination to enjoy food consumption and the individual’s concern with the long-term adverse effect of physiologically improper levels of food consumption on her, or his, health and probability of survival. However, the model’s prediction of diverging spiral trajectories of food consumption and weight is unlikely to be supported by empirical analyses. It is, of course, a prediction of a stylised model. As stressed from the outset, the model does not take into account psychological and physiological problems that may explain cases of obesity and anorexia, and environmental disasters that might lead to involuntary periods of starvation and loss of weight. Nor it takes into account social and cultural norms that may affect the stationary levels of the rational food consumption and weight and dampen the oscillations of the rational food consumption and weight. A modification of the model to include some possible social and cultural aspects is provided in the following section.

5. The effect of social and cultural preferences on weight
Overweightness and underweightness might also stem from conforming to social and cultural norms and perceptions. For instance, in some oriental countries overweightness is associated with beauty, contentment, grace and wisdom, whereas in some occidental countries slenderness is much more appreciated. If this associations between physical attractiveness and weight is valid it is expected that, ceteris paribus, oriental slims and occidental fats are the less fortunate people in their domestic markets of partners. It is shown in the following, however, that despite their failure to conform to the socio-cultural norms of physical appearance, their weight is closer to the socio-culturally preferred weight than otherwise.
The socio-culturally preferred weight can be incorporated into the basic model by assuming that the individual suffers from a loss of utility by not conforming to the socio-culturally preferred weight \( W^{sp} \). In this case, the rational food-consumer’s objective can be rendered as:

\[
\max_{c} \int_{0}^{T} e^{-\rho t} U(c(t), (W(t) - W^{sp})^2) \Phi((W(t) - W^*)^2) dt
\]  

subject to a weight-motion equation (3). Following the solution procedure described in section 2, the necessary conditions for maximum and the associated singular control equation imply that the rationally optimal food-consumption and weight trajectories should satisfy the no-arbitrage rule:

\[
\frac{U_{cc}}{U_c} \dot{c} + \left[ \frac{U_{cW}}{U_c} + \frac{\Phi_w}{\Phi} \right] \dot{W} - \left[ \frac{U}{U_c} \frac{\Phi_w}{\Phi} + \frac{U_w}{U_c} \right] = \rho + \delta
\]  

(30)

and that in steady state

\[
- \left[ \frac{U}{U_c} \frac{\Phi_w(W_{ss}')}{\Phi(W_{ss}')} + \frac{U_w(c_{ss}', W_{ss}')}{U_c(c_{ss}', W_{ss}')} \right] = \rho + \delta.
\]  

(31)

In order to compare the stationary weight in this case \( W_{ss}' \) to the stationary weight analysed in section 3 where no socio-cultural pressure exists \( W_{ss} \), the probability of living beyond \( t \) is specified as depicted earlier by equation (16) and the instantaneous utility from eating described by equation (17) is deflated by the extent of not conforming to the socio-culturally preferred weight as follows:

\[
U(t) = \frac{e^\beta}{(W(t) - W^{sp})^2}.
\]  

(32)

These specifications imply that the stationary weight when there is a socio-culturally preferred weight should satisfy:

\[
W_{ss}' [\mu(W_{ss}' - W^*) + (W_{ss}' - W^{sp})^{-1}] = 0.5 \beta (\rho + \delta) / \delta
\]  

(33)

whereas the stationary weight when there is no socio-culturally preferred weight should satisfy:

\[
W_{ss} [\mu(W_{ss} - W^*)] = 0.5 \beta (\rho + \delta) / \delta.
\]  

(34)
If $W^\prime_{ss} > W^{sp}_{ss}$, which is the likely characteristic of occidental people who are less successful in the matching contest, then $W^\prime_{ss} < W_{ss}$. That is, when there exists a socio-cultural norm of appearance their stationary weight is lower than otherwise. If, however, $W^\prime_{ss} < W^{sp}_{ss}$, which is the likely case of oriental people who are less successful in the matching contest, then $W^\prime_{ss} > W_{ss}$. That is, when there exists a socio-cultural norm of appearance their stationary weight is greater than otherwise.

The increased complexity of the no-arbitrage rule made it impossible to obtain a closed-form solution to the stationary weight and to analyse the behaviour of the food-consumption and weight trajectories in this case.

6. Conclusion

The paper presented a manageable stylised basic model of non-addictive eating that can explain overweightness, underweightness and cyclical food consumption by assuming that the satisfaction from eating is counterbalanced by increasing probability of dying as weight deviates from the physiologically optimal level. It was found that when physiological, psychological, environmental and socio-cultural reasons for divergence from a physiologically optimal weight do not exist, the steady state for a lifetime expected-utility maximiser is a state of overweightness. The rationally optimal stationary level of overweightness was shown to be rising with the individual’s rate of time preference and elasticity of utility but declining with his, or her, rate of calories burning and rate of decline of the probability of continuing living caused by an infinitesimal rise in the quadratic deviation from the physiologically optimal weight. It was shown, however, that even a small initial deviation from this rationally optimal stationary weight is followed by explosive oscillations, i.e., binges and strict diets, and possibly a severe and chronic underweightness in a late stage of life. The incorporation of socio-cultural norms into the basic model revealed that when there exists a socio-cultural norm of appearance the stationary weight of fat people is lower than otherwise and the stationary weight of lean people is greater than otherwise.

Some other possible effects of deviation from the physiologically optimal weight and budget considerations were not incorporated into the model for sake of simplicity. In a
broader framework, instantaneous utility may be presented as derived from eating and devoting a fraction \( (\ell) \) of an instance to leisure activities (the rest, \( 1 - \ell \), is devoted to income-generating activities), and a deviation from the physiologically optimal weight might also lead to a loss of satisfaction from leisure activities and a loss of efficiency in generating income. Intuitively, it is expected that the incorporation of these elements would reduce the difference between the rationally optimal stationary weight and the physiologically optimal weight, dampen the oscillations of the food-weight joint trajectory, and increase the likelihood of convergence to the stationary weight. An extension of the basic model along these lines is described in Appendix B. However, the complexity of the extended model rendered the assessment of this intuitive assertion and the properties of the optimal trajectories of food consumption, leisure and weight impossible.

**APPENDIX A: An Explanation of the Transition from Equation (1) to Equation (2)**

Let \( F(t) \) be the cumulative density function associated with the probability of dying at \( t \) (i.e., the probability of living up to \( t \)). Then

\[
p = F'(t) \tag{A1}
\]

and equation (1) can be rendered as

\[
J = \int_0^T F'(t) \left[ \int_0^t e^{-\rho \tau} U(c(\tau))d\tau \right] dt = \int_0^\tau v(t)du \tag{A2}
\]

where,

\[
v = \int_0^t e^{-\rho \tau} U(c(\tau))d\tau \tag{A3}
\]

and

\[
u = -(1 - F(t)) \tag{A4}
\]

The integration by parts rule suggests that

\[
J = \int_0^\tau vdu = uv - \int_0^\tau udv \tag{A5}
\]
Note, however, that
\[ uv = \left[ (1 - F(t)) \int_{0}^{t} e^{-\rho \tau} U(c(\tau)) d\tau \right]_{0}^{\tau} = 0 \]  
(A6)
because when evaluated at the lower limit
\[ uv = \left[ (1 - F(0)) \int_{0}^{0} e^{-\rho \tau} U(c(\tau)) d\tau \right] = 0 \]  
(A7)
and when evaluated at the upper limit
\[ uv = \left[ (1 - F(T)) \int_{0}^{T} e^{-\rho \tau} U(c(\tau)) d\tau \right] = 0 \]  
(A8)
as
\[ F(T) = 1. \]  
(A9)
Hence,
\[ J = -\int_{0}^{\tau} uv. \]  
(A10)
By virtue of equation (A3)
\[ dv = e^{-\rho \tau} d\tau \]  
(A11)
and the substitution of equations (A4) and (A11) into (A10) implies
\[ J = \int_{0}^{\tau} e^{-\rho \tau} U(c(t)) \Phi(t) dt \]  
(A12)
where
\[ \Phi(t) \equiv -u(t) = 1 - F(t) \]  
(A.13)
and indicating the probability of living at least until (or alternatively put, beyond) \( t \).

**APPENDIX B: An Extension of the Basic Model**

An intensive effort has been made in extending the basic model. By incorporating the elements indicated in the concluding section into the basic model, the modified decision problem of a rational food-consumer can be portrayed as follows:
\[
\max_{\{c, \ell\}} \int_0^T e^{-\mu t} U(c(t), [1 - \gamma(W(t) - W^*)^2] \ell(t)) \Phi((W(t) - W^*)^2) dt
\]  

(B1)

where weight is gained by consuming food and is lost by spending energy on work and leisure activities and proportionally to its current level:

\[
\dot{W}(t) = c(t) - [\delta_1 \ell + \delta_2 (1 - \ell)] W(t)
\]  

(B2)

and where

\[
c(t) = \left[1 - \theta(W(t) - W^*)^2\right] f(1 - \ell(t)).
\]

(B3)

In this framework, \(\delta_1\) and \(\delta_2\) are positive scalars indicating the marginal effect of weight on burning calories, and subsequently weight, in leisure activities and work, respectively; \(\gamma\) and \(\theta\) are positive scalars indicating the marginal effects of being overweight or underweight on satisfaction from leisure activities and on income generation, respectively; and \(f\) is a concave earning function \((f' > 0\) and \(f'' < 0\)) for a person having physiologically optimal weight.

Finally, the degree of complexity was largely increased when equation (16) was modified as follows:

\[
\Phi = \Phi_0 e^{-\mu \frac{(W(t) - W^*)^2}{T-t}}
\]

(B4)

to incorporate the possible effect of age and the moderating influence of the distance from the upper-bound on life expectancy on the adverse effect of a deviation from the physiological weight on the probability of living beyond \(t\).
References


