On the rationality of eating junk food

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Abstract

Rational junk-food consumption is analyzed within an expected lifetime utility maximizing framework in which longevity and productivity rise with health, and health deteriorates with junk-food consumption. As long as the junk food’s relative taste-price differential is positive, the consumer’s rational diet deviates from the physiologically optimal one and generates lower than maximal levels of health and productivity. The stationary junk-food consumption, health and productivity depend on the consumer’s tastes, prices, endurance, appetite and time preferences. Damped, or explosive, cycles of junk-food consumption and health may reflect a repulsive, or addictive, effect of ingredients excessively contained in junk food, respectively.

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1. Introduction

Attempts to explain the prevalence of overweight, obesity and their associated health problems have usually considered a time-value induced reliance on commercially supplied meals, technological developments that made meals cheaper and work more sedentary, and an inverse relationship between eating and smoking (cf. Philipson and Posner, 1999; Ruhm, 2000; Lakdawalla and Philipson, 2002; Cutler et al., 2003; Cawley et al., 2003; Chou et al., 2004). Some other attempts have highlighted the role of behavioural factors. Komlos et al. (2002) have argued that a rise in the rate of time preference contributes to the prevalence and intensity of overweightness and obesity. O'Donoghue and Rabin’s (1999, 2000) argument of time-inconsistent preferences and immediate gratification can add significance to the role of strong time preferences in explaining excessive eating and, in turn, overweight and obesity. Levy (2002) has argued that overweight is a rationally optimal steady-state, increasing in the individual’s rate of time preference, in the elasticity of the utility derived from food-consumption and in the degree of sedentary behavior, yet decreasing in mortality risk and responsive to cultural norms.

Complementing the above list of possible causes of overweight and obesity is the sort of food eaten. The significance of this factor is possibly reflected in empirical studies by the large positive effect of the per capita number of restaurants (of which many specialize in fast food and snack food) on the body mass index and the probability of being obese (cf. Chou et al., 2004). Food can be classified as healthy food or less healthy food. The paper refers to the latter as junk food—a slang term widely used in medical, nutritional and public health research (cf. Bernardis and Bellinger, 1991; Herr-Wagner et al., 1999; Dixon et al., 2007; Brigelius-Flohé and Davies, 2007). The distinction between healthy food and junk food depends on the
concentration of ingredients such as fat, salt, sugar and food additives (e.g., monosodium glutamate and tartrazine), whose presence in the human body beyond a critical level is harmful. Junk food contains large quantities of these ingredients, while healthy food contains recommended amounts. Furthermore, junk food is lacking vital nutrients such as fibers, proteins and vitamins.¹

The size of the fast-food and snack-food industries and the prevalence of overweight and obesity suggest that the actual diets of many, not necessarily irrational, consumers substantially deviate from the physiologically optimal abstinence from junk food consumption. There can be a difference between the physiologically optimal choice and the rational choice. The consumer’s rational choice of food is the focus of this paper. Taking a rational consumer to be a maximiser of expected lifetime utility subject to the effect of food-composition on the evolution of his health, the paper asks whether junk food can be a component of this consumer’s diet.

Levine et al.’s (2003) study on the neurobiology of preference has shown that central regulatory mechanisms favor foods containing sugar and fat over other nutrients. Having a higher concentration of these ingredients than its healthy substitute makes junk food taste-wise appealing for rational consumers. Herr-Wagner et al. (1999), Smith (2004) and Dixon et al. (2007) have analyzed the manipulative nature of advertisements of fast and snack foods.

Furthermore, several studies, including Drenowski (2003), Lakdawalla and Philipson (2002) and Philipson and Posner (1999), have determined that junk food is often less expensive than healthy food due to cheaper ingredients, easier preparation

¹ A strategy known as nutrient profiling that sets limits on the quantities of the aforementioned ingredients helps regulatory agencies identify junk food. Recommended dietary allowances and adequate intakes organized by age and gender can be found in the Dietary Recommended Intakes Tables for vitamins, minerals and macronutrients, which are published by the Food and Nutrients Board, Institute of Medicine, National Academy of Sciences, USA.
process and storage, and value of time. There are claims that the taste and price advantages of junk food for certain consumers have been exploited by some of the least expected institutions. Anderson and Butcher (2005) have argued that, due to budgetary reasons, the availability of junk foods in schools has been increased, and that the greater availability explains about one-fifth of the increase in average body mass index among adolescents in the United States over the last decade.

Risk differential may also affect consumers’ choice of diet. Nutritionally inadequate diet constitutes a health hazard. A number of studies, including Deolalikar (1988) and Alderman et al. (2005), have demonstrated the link between chronic malnutrition and loss of productivity. Mialon and Mialon (2005) have argued that the availability of a less harmful substitute to a harmful substance (e.g., light beer and light cigarettes) does not necessarily improve the consumer’s health. It reduces the risk for people with high taste for the more harmful good. However, it increases the risk for people with a sufficiently low taste for the more harmful good due to a large consumption of the less harmful good, which still contains significant quantities of harmful substances. Unlike Mialon and Mialon (ibid), the junk food’s healthy substitute in the present analysis contains insignificant quantities of harmful substances. Also the less appealing flavor and high price of healthy food, as well as the high level of health awareness of its consumers, prevent excessive intakes.

For non-myopic consumers the effects of the short-term taste and price advantages of junk food on its level of consumption are moderated by the implications of its harmful ingredients for health and productivity. The analysis of the rational consumer’s choice of junk-food and healthy-food consumption and its implications for health and productivity is conducted by incorporating the possible short-term advantage and the long-term disadvantage of junk food into an expected lifetime
utility maximization model. Rational food consumers are assumed to have self-control and time-consistent preferences. In addition to the effects of junk-healthy foods’ price, taste and risk differentials, the analysis explores the effects of consumers’ time preferences, appetite and endurance on rational dietary choices.

The dynamic model of rational dietary choice developed in this paper distinguishes between a physiologically optimal dietary path and a rational dietary path. The physiologically optimal dietary path excludes junk food and leads to maximum health, productivity and life-expectancy but not necessarily to maximum lifetime utility. A rational dietary path is the continuum of combinations of healthy food and junk-food that maximizes the individual’s lifetime expected utility from eating subject to the individual’s health-transition equation. In the case of consumers with a negative relative taste-price differential between junk food and healthy food, abstinence from junk food is rational and the physiologically optimal dietary path is also optimal from the perspective of expected lifetime utility. In the case of consumers with a positive relative taste-price differential, some indulgence in junk food maximizes expected lifetime utility, but leads to lower than maximal health, productivity and longevity. In the latter case, the consumers’ steady-state health and productivity levels depend on the size of the consumers’ relative taste-price differential, on the strength of their time-preferences relative to their intrinsic endurance-appetite ratio, and on the health-depreciating effect of junk food. The analysis suggests that as long as the consumer’s time-preference rate is sufficiently low, the consumer’s health and junk-food consumption oscillates around the steady state. The consumer’s health and productivity oscillations diminish with the passage of time if her current junk-food consumption decelerates her future junk-food consumption. However, the health and productivity oscillations intensify with the passage of time if the current junk-food
consumption accelerates future junk-food consumption. A junk-food consumption tax that is greater than the relative taste-price differential can steer health and productivity of rational consumers to the maximal levels. However, the outcome is not Pareto-superior to the free-market one that includes cases of partial and total indulgence.

2. Diet, utility, health, productivity and endurance

The model constructed in this section for displaying the relationships between diet, utility, health, productivity and endurance includes, for simplicity, only two goods—healthy food and junk food. A junk-food unit is denoted by \( j \) and a healthy-food unit by \( h \). The consumer’s price ratio of the junk-food unit and the healthy-food unit (henceforth, relative price) is taken to be a time-invariant \( p \) and, in agreement with the arguments and empirical findings indicated in the introduction, \( p < 1 \). It may vary from one consumer to another in accordance with their local market conditions, food preparation skills and value of time. The consumer’s taste-ratio of the junk-food unit and the healthy-food unit (henceforth, relative taste) is a positive scalar \( \alpha \) reflecting time-consistent tastes. For consumers with high taste for junk food, \( \alpha > 1 \). For consumers with low taste for junk food, \( \alpha < 1 \). The number of healthy-food units required for maintaining the consumer in perfect health at any instant \( t \) is a positive scalar \( c_h^0 \), whereas the number of junk-food units required for maintaining the consumer in perfect health is nil (i.e., \((c_h^0, 0)\) is the physiologically optimal diet for a perfectly healthy consumer). The actual number of healthy-food units eaten by the consumer at \( t \) is \( c_h(t) \in (0, c_h^0) \). The actual number of junk-food units eaten by the consumer at \( t \) is \( c_j(t) \geq 0 \). The consumer’s actual diet at \( t \) is \((c_h(t), c_j(t))\).
The consumer’s satisfaction from her diet at \( t \) is represented by a function \( u(c_h(t), c_j(t)) \) that has the following properties. Food is essential: \( u(0,0) = 0 \). Yet neither health food nor junk food is essential by itself: \( u(0,c_j), u(c_h,0) > 0 \). The marginal instantaneous satisfaction with respect to each type of food is positive and diminishing: \( u_h, u_j > 0 \) and \( u_{hh}, u_{jj} < 0 \). Healthy food and junk food are substitutes: \( u_{hj} < 0 \). For any \( c_j = c_h, u_j > u_h \) if \( \alpha > 1 \), \( u_j = u_h \) if \( \alpha = 1 \), or \( u_j < u_h \) if \( \alpha < 1 \).

Consistent with these assumptions, the following convenient explicit form is used:

\[
\begin{align*}
  u_t &= [\alpha c_j(t) + c_h(t)]^\beta, \\
  \text{where } \alpha c_j + c_h &= \text{interpreted as the taste-adjusted meal at } t \text{ and the scalar } 0 < \beta < 1 \text{ as the eating-satisfaction elasticity with respect to the taste-adjusted meal}.^2
\end{align*}
\]

It reflects the consumer’s ability to enjoy eating. A consumer who considers eating a mere physiological necessity has a small \( \beta \), whereas a consumer who regards eating also as a pleasurable activity has a large \( \beta \). We refer to \( \beta \) as the consumer’s intrinsic appetite.^3

The consumer’s health, \( x(t) \in (0,1) \), determines her productivity—the extent to which the consumer realizes her full-capacity income. Productivity is maximal when the consumer is perfectly healthy \( (x=1) \) and converges to zero as the consumer becomes terminally sick \( (x=0) \). Skill and employment opportunities determine the

\[\text{Footnotes:}^2 \text{ The ratio of the satisfaction-elasticities with respect to junk-food and healthy food is equal to the product of the relative taste and quantities, } \alpha(c_j/c_h).^3 \text{ As appetite may diminish with eating, the adjective } intrinsic \text{ is added to the description of the consumer’s appetite coefficient } \beta. \text{ This constant can also be interpreted as the ratio of the eating-satisfaction elasticity with respect to healthy food } (\xi_h) \text{ and the share of healthy food in the taste-adjusted meal (i.e., } \beta = \xi_h/[c_h/(\alpha c_j + c_h)].\]
consumer’s full-capacity income, which is taken for simplicity to be a positive scalar \( \hat{y} \). Hence, the consumer’s instantaneous income is given by:

\[
y(t) = x(t) \hat{y}
\]  

(2)

and, with the full price of healthy food being a numeraire, the consumer’s instantaneous budget constraint is given by:\(^4\)

\[
p c_j(t) + c_h(t) = x(t) \hat{y}.
\]  

(3)

The right-hand side of the budget constraint reflects the assumption that the healthier the consumer, the greater her spending on food. This is due to a higher share of the more expensive healthy food in the diet and a greater work effort.

Health is improved by reducing the consumption of junk food and increasing the consumption of healthy food. Once reached, perfect health \((x = 1)\) is maintained by adhering to the physiologically optimal diet \((c_h^0, 0)\). Correspondingly, and in recalling Eq. (2), the balanced-budget equation requires that the full-capacity income earned by a perfectly healthy consumer is equal to the cost of her physiologically optimal diet. That is, \( \hat{y} = c_h^0 \). To let perfect health be attainable, this equality is assumed. With this assumption, the instantaneous budget constraint can be rewritten as:

\[
c_h(t) = x(t)c_h^0 - p c_j(t)
\]  

(4)

and, by substitution into Eq. (1), the instantaneous utility function can be further expressed as:

\[
u_t = [(\alpha - p)c_j(t) + x(t)c_h^0]^{\beta}.
\]  

(5)

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\(^4\) The presentation of the more general case of intertemporal-budget constraint with borrowing and lending requires the inclusion of an extra state variable (outstanding debt or credit) and interest rate. The consideration of such intertemporal budget constraint complicates the analysis tremendously while not being a major issue.
The consumer’s health is deteriorated by the deviation from the physiologically optimal diet and improved by a natural recovery process. The instantaneous change in the consumer’s health is represented by a logistic function displaying a diminishing relative health-improvement rate in junk-food consumption, a diminishing health-improvement rate \( (r) \) in the level of health, and a unit upper bound and a zero lower bound on the consumer’s health. Using \( c_j(t)/c_h^0 \) as an index of the excessive physiological inadequacy of the actual diet, the evolution of the consumer’s health is given by:

\[
\dot{x}(t) = \left\{1 - \delta (c_j(t)/c_h^0)\right\}[1 - x(t)]x(t),
\]

where \( \delta \) is a positive scalar indicating the marginal adverse effect of the physiologically inadequate diet on the relative rate of improvement of the consumer’s health. When junk food is avoided the current recovery rate \((\dot{x}(t)/x(t))\) is maximal and equal to the recovery rate \(1 - x(t)\) facilitated by the currently affordable healthiest diet. The interpretation of the health-motion equation is enhanced by noting that

\[
1 - \delta (c_j(t)/c_h^0) = [\dot{x}(t)/x(t)]/[1 - x(t)]. \tag{6'}
\]

Namely, \(1 - \delta (c_j(t)/c_h^0)\) is the current rate of health-change \((\dot{x}(t)/x(t))\) relative to the currently maximal recovery rate \((1 - x(t))\). This current _relative health-change rate_ is hindered by current junk-food consumption and is negative for \(c_j(t) > c_h^0/\delta\).\(^5\)

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\(^5\) The case of a negative relative health-improvement rate does not violate the assumption that \(x\) lies within the (positive) unit interval as long as the initial value of \(x\) is smaller than 1. Furthermore, when \(x\) is close to zero and the consumption of junk food is lower than \(c_h^0/\delta\), \(1 - \delta (c_j(t)/c_h^0)\) can be interpreted as the junk-food weakened recovery rate from a near-death situation. Had the healthy food contained significant quantities of harmful ingredients, the Mialon and Mialon’s (2005) proposition could be reproduced by specifying the health-motion equation as \(\dot{x}(t) = [1 - \delta(c_j(t) + \mu c_h^0(t))][1 - x(t)]x(t)\), where \(0 < \mu < 1\) indicates the harm caused by consuming a unit of healthy food relative to that caused by a unit of junk food.
The consumer’s probability of survival to the next instant rises with her health. It is equal to one when the consumer is perfectly healthy \( (x = 1) \) and converges to zero as the consumer’s health diminishes. For tractability, it is taken to be isoelastic. In formal terms, let \( F(t) \) be the cumulative distribution function associated with the probability density function \( (\phi(t)) \) of dying at \( t \), then \( \Phi(t) = 1 - F(t) \) is the probability of living beyond \( t \) (i.e., endurance probability). It is assumed that \( \Phi(t) = \Phi(x(t)) \) with \( \Phi_x > 0 \) (a positive health effect), \( \lim_{x \to 1} \Phi = 1 \) and \( \lim_{x \to 0} \Phi = 0 \). It is further assumed that the elasticity of the consumer’s endurance probability \( (\Phi_x \frac{\dot{x}}{\Phi}) \) is equal to a positive scalar \( \eta \). Namely,

\[
\Phi(t) = x(t)^\eta,
\]

(7)

where \( 0 \leq \Phi \leq 1 \) for any \( \eta > 0 \) since \( 0 \leq x \leq 1 \). Consequently, the rate of change of the endurance probability is proportional to the rate of change of health: \( \frac{\dot{\Phi}}{\Phi} = \eta \frac{\dot{x}}{x} \). We refer to the coefficient \( \eta \) as the consumer’s intrinsic endurance.\(^6\)

3. Rational choice

Let us consider a consumer who has a fixed, positive rate of time preference \( (\rho) \) and an additively separable lifetime utility function, and who chooses her dietary path so as to maximize her expected lifetime utility from food consumption over the remainder of her life subject to her health motion equation. Since the time of death is random, the consumer multiplies her accumulated utility from food consumption between the starting point of her planning horizon and her possible time of death

\(^6\) As the prospects of endurance increase with health, the adjective intrinsic is added to the description of the endurance-probability’s coefficient.
by the probability of dying at that instant \((\phi(t))\). The products of \(\phi(t)\) and \(\int_0^t e^{-\rho \tau} u_\tau d\tau\) associated with any possible life expectancy \(0 \leq t \leq \infty\) are considered by the consumer. The sum of all these products, \(\int_0^\infty \phi(t) \int_0^t e^{-\rho \tau} u_\tau d\tau dt\), is the consumer’s expected lifetime utility \((E(V))\). Through integrating by parts and in recalling Eq. (7):\(^7\)

\[
E(V) = \int_0^\infty \Phi(x(t)) e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} x(t)^\eta u_t dt.
\] (8)

The right-hand-side term of Eq. (8) provides an alternative interpretation of the consumer’s expected lifetime utility—one that is based on the association of quality of life and health. The number of quality-adjusted life-years is used in cost-benefit analysis of health-investment projects as an index of well-being. It combines the duration of life and health condition into a single utility index (cf. Bleichrodt, 1995; and Bleichrodt and Quiggin, 1999). Likewise, \(0 \leq x(t)^\eta \leq 1\) can be alternatively viewed as the consumer’s life-quality index and \(\int_0^\infty e^{-\rho t} x(t)^\eta u_t dt\) as the consumer’s quality-adjusted lifetime-utility from food consumption.

Substituting Eq. (5) into Eq. (8) for \(u_t\), the rational junk-food consumption path is found by \(\max \left\{ e^{-\rho t} x(t)^\eta [(\alpha - p)c_j(t) + x(t)c_h^o] \right\} dt \) subject to the health-motion equation (6).

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\(^7\) See Levy (2002a, 2002b) for proof and use of \(\int_0^\infty \phi(t) \int_0^t e^{-\rho \tau} u_\tau d\tau d\tau dt = \int_0^\infty \Phi(t)e^{-\rho t} u_t dt\) for analyzing the prevalence of overweightness and HIV-AIDS among rational people.
4. Rational abstinence or indulgence

When analyzing junk-food consumption the polar phenomena of abstinence and indulgence deserve attention. Abstinence is usually attributed to dogma and/or a life-threatening physiological problem, whereas indulgence is usually due to loss of self-control. In the context of the present optimal-control problem, abstinence or indulgence may arise as corner solutions for consumers with certain pecuniary values of time and rates of time-preferences.

If the relative price of junk food exceeds the relative taste of junk food \( p > \alpha \), a diet that is free of junk food is rationally optimal, converges to the utmost physiologically optimal one \((c^0_h)\), and maximizes the consumer’s health and productivity. The underlying rationale is as follows. Recalling Eq. (5), \( \alpha < p \) implies that the instantaneous utilities from consuming junk food are negative. Hence, a consumer who maximizes \( \int_0^\infty e^{-t\alpha} x(t)^\eta u_t dt \) must maintain a diet free of junk food: \( c_h(t) = y(t) = x(t)\hat{y} \) every instant \( t \). In which case, and in recalling Eq. (6),

\[
\lim_{t \to \infty} x = 1. \quad \text{In turn, and recalling Eq. (2),} \quad \lim_{t \to \infty} y = \hat{y}. \quad \text{Recalling Eq. (3) and Eq. (2),}
\]

\[
\lim_{x \to 1} \hat{y} = c^0_h. \quad \text{Hence,} \quad \lim_{t \to \infty} c_h = c^0_h.
\]

This rational abstinence implies that junk food does not necessarily constitute the diet of low-income consumers. Suppose that consumers have similar tastes. Since the pecuniary value of time is higher for middle-income consumers than for lower-income consumers, it is likely that the formers frequent fast-food and snack-food restaurants and that the latter abstain. Indeed, junk food was significantly introduced to major cities in developing countries by foreign fast-food and snack-food companies when it became affordable to middle-income consumers. Furthermore, the pecuniary value of
time for high-income consumers is high, but they can afford to employ a cook for home meals and/or eat in high quality restaurants. By doing so they increase their quality-adjusted life years. Hence, there may be an inverted U-shaped relationship between the probability of abstinence and income.

In contrast, if the relative taste of junk food exceeds the relative price of junk food (i.e., \( \alpha > p \)) which, recalling that \( p < 1 \), also can be the case when healthy food is tastier than junk food, \( \alpha < 1 \) and the consumer is myopic (\( \rho \to \infty \)), a diet that is entirely based on junk food is rationally optimal, but maximizes the consumer’s loss of health and productivity and ultimately leads to complete self-destruction. This rational indulgence is consistent with O’Donghue and Rabin’s (1999 and 2000) immediate gratification stemming from time-inconsistent preferences. When \( \alpha > p \) and \( \rho \to \infty \), the marginal instantaneous satisfactions from the junk food are positive and only the present utility matters to the consumer. The value of future health to the consumer is nil and hence \( c_h(t) = 0 \) and \( c_j(t) = x(t)\hat{y}/p \) every instant. Recalling Eq. (6), \( \lim_{t \to \infty} x = 0 \) and, in turn and recalling Eq. (2), \( \lim_{t \to \infty} y = 0 \).

5. Rational composite diet and the value of health

The analysis of the consumer’s rational choice of junk food and healthy food composition continues under the assumptions of positive relative taste-price differential (\( \alpha - p > 0 \)) and non-myopia. The Hamiltonian corresponding to the consumer’s constrained maximization problem portrayed in section 3 is:

\[
H = e^{-\rho t} x^\beta [(\alpha - p)c_j + x c_h^o]^{\beta} + \lambda [1 - \delta (c_j / c_h^o)](1 - x)x
\]

where the co-state variable \( \lambda \) indicates the shadow present value of the consumer’s health. (The time-index is omitted for tractability.) Since \( 0 < \beta < 1 \) the Hamiltonian is
concave in $c_j$. However, neither $e^{-\rho x^\eta[(\alpha - p)c_j + xc_h^o]} \beta$ nor $[1 - \delta(c_j / c_h^o)](1 - x)x$
is necessarily concave in the state variable ($x$). In turn, the Hamiltonian is not
necessarily concave in $x$. In which case, Mangasarian’s theorem on the sufficiency of
Pontryagin’s maximum-principle conditions is not valid. Non-concavity of a
Hamiltonian in its state variable plays a crucial role in generating unstable steady
states and, possibly, a Dechert-Nishimura-Skiba point (cf. Clark, 1971; Skiba, 1978;
Dechert and Nishimura, 1983).

In addition to the state-equation (6), maximum expected lifetime satisfaction from
food requires that the change in the consumer’s valuation of health is given by:

$$\dot{\vartheta} = \left[\eta x^\eta Z^\beta + x^\eta \beta Z^\beta \eta c_h^o \right] e^{-\mu x} - \lambda (1 - 2x)(1 - \delta c_j / c_h^o)$$

and that along the rational food-consumption path the marginal satisfaction from
eating junk food, discounted by both the consumer’s time preferences and prospects
of endurance, is equal to the value of the marginal loss of health caused by eating junk
food:

$$x^\eta e^{-\mu x} \beta Z^{-1}(\alpha - p) - \lambda (\delta / c_h^o) x(1 - x) = 0$$

where $Z \equiv (\alpha - p)c_j + xc_h^o$.

Value increases with scarcity—the poorer the consumer’s health condition, the
greater her appreciation of health. The first term on the right-hand side of Eq. (10)
reveals the negative combined effect of the rise in the probability of endurance and
income stemming from an infinitesimal improvement in health on the consumer’s
evaluation of her health. The second term represents the effects of an infinitesimal
improvement of health on further improvement in health, $\dot{x}$. As implied by the
logistic-health improvement function, up to a critical level of health (e.g., 0.5 had junk
food been avoided) $\dot{x}$ increases with $x$, due to strong natural recovery, and then
decreases. A large (small) health improvement decelerates (accelerates) the consumer’s evaluation of health. Eq. (10), in conjunction with the optimality condition (11), implies further that along the rational junk-food consumption path the rate of change of the shadow value of health is given by:

\[
\frac{\dot{\lambda}(t)}{\lambda(t)} = \left[ (\eta / \beta) c_j + [1 + (\eta / \beta)] \frac{xc_h^0}{\alpha - p} \right] \left( \frac{\delta / c_h^0 (1 - x) - (1 - 2x) [1 - (\delta / c_h^0) c_j]}{\delta / c_h^0} \right). \tag{12}
\]

In turn, the effect of junk-food consumption on the evaluation of health is:

\[
\frac{\partial (\dot{\lambda} / \lambda)}{\partial c_j} = \left[ \frac{\eta}{\beta} - 2 \right] x - \left( \frac{\eta}{\beta} - 1 \right) \frac{\delta}{\beta c_h^0}.
\tag{13}
\]

Eq. (13) implies that if \( \eta \geq \beta \) then \( \frac{\partial (\dot{\lambda} / \lambda)}{\partial c_j} < 0 \) for every \( x \). That is, as long as the intrinsic endurance is not smaller than the intrinsic appetite an increase in the intake of junk food decelerates the consumer’s evaluation of her health. A strong intrinsic appetite leads to a large consumption of the tastier and cheaper junk food and thereby to poor health. Poor health gives rise to self concerns about, and appreciation of, health. However, having at least as strong intrinsic endurance gives rise to complacency. Eq. (13) also implies that if \( \eta < \beta \) then \( \frac{\partial (\dot{\lambda} / \lambda)}{\partial c_j} > 0 \), as \( x \leq \left( \frac{1 - \eta / \beta}{2 - \eta / \beta} \right) \).

That is, when the intrinsic endurance is smaller than the intrinsic appetite, an increase in junk-food consumption accelerates the consumer’s evaluation of health if her health is below a critical level, \( \left( \frac{1 - \eta / \beta}{2 - \eta / \beta} \right) \). The lower the intrinsic endurance-appetite ratio the higher the critical health level.

As explained in greater detail in Appendix A, the instantaneous change in the rationally self-controlled junk-food consumption is given by:
\[ \dot{c}_j = \frac{\{(\alpha - p)c_j + xc_h^o\}}{(1 - \beta)(\alpha - p)} \rho - \frac{(\eta/Z + c_h^o)(1 - x)(\delta/c_h^o)x}{(\alpha - p)} - \frac{\eta - (1 - \beta)c_h^o}{Z} \dot{x}. \] (14)

The system comprising Eq. (14) and Eq. (6) portrays the joint evolution of the rationally junk-food consumption and health. Interior steady states (SS) are analyzed in the following section for exploring the possible long-run levels of rational junk-food consumption and health.

6. Stationary levels and cycles of junk-food consumption and health

In steady state the junk-food consumption is \( c_{j,ss} = c_h^o / \delta \) and, as shown in Appendix A, the consumer’s health and productivity levels are:

\[ x_{ss,1,2} = 0.5 \left\{ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right\} \pm \sqrt{\left\{ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right\}^2 + \frac{4(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}}. \] (15)

Intuitively, one may argue that the stationary levels of health and, in turn, productivity are adversely affected by the consumer’s rate of time preference. Consumers endowed with high rate of time preference are strongly attracted by the price advantage, and some also by a taste advantage, of junk food over healthy food and attach a low weight to the long-term adverse effect of junk-food consumption on their health, productivity and longevity. Junk food comprises a large portion of the regular diet of these consumers and severely hinders their health, productivity and endurance. Eq. (15) reveals a more complex relationship. The possibility of multiple steady states and

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8 It can be shown that

\[ x_{ss,1,2} = 0.5 \left\{ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta/c_h^o} \right\} \pm \sqrt{\left\{ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta/c_h^o} \right\}^2 + \frac{4(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta/c_h^o}}. \]

The exclusion of \( \dot{y} \) and \( c_h^o \) from Eq. (15) is due to \( \dot{y} = c_h^o \), which is implied the assumptions that there are only two goods, that the budget is instantaneously balanced, and that \( \dot{y} \) and \( c_h^o \) are, respectively, the income and diet of a perfectly healthy consumer.
stationary health condition depend on the size of the consumer’s rate of time preference relative to the consumer’s intrinsic endurance-appetite ratio, which affects the location of the U-shaped $\dot{c}_j = 0$ isocline. The greater the difference between the rate of time preference and the intrinsic endurance-appetite ratio, the more upwardly located the $\dot{c}_j = 0$ isocline and its intercept in the $x - c_j$ plane. (See Appendix B.)

If the consumer’s rate of time preference is equal to her intrinsic endurance-appetite ratio ($\rho = \eta / \beta$), there exists, as displayed by Figure 1, two stationary states of health—a corner one, $x_{ss} = 0$, and an interior one, $x_{ss} = 1 - [(\alpha - p)\eta / (\eta + \beta)\delta]$. The existence of the interior steady state requires that $(\eta + \beta)\delta$ is sufficiently large so that $(\alpha - p)\eta / (\eta + \beta)\delta < 1$. With regard to the interior steady state, the greater the intrinsic appetite and the junk food’s health-depreciating effect ($\delta$), the better the consumer’s stationary health and productivity, whereas the greater the junk and healthy foods’ relative taste-price differential ($\alpha - p$) and the intrinsic endurance, the worse the consumer’s stationary health and productivity. The latter part of this argument reflects a complacency effect of the intrinsic endurance. When a slight improvement in health generates significant extension of life expectancy, a higher degree of leniency toward the consumption of the less healthy, yet cheaper and possibly tastier, type of food is entertained by a rational consumer. This complacency is manifested in bad health and, in turn, low level of productivity. The other comparative-static results associated with the interior steady state indicate that a large taste-price differential encourages junk-food consumption and thereby maintenance of poor health. Interestingly, poor health can also be associated with a weak intrinsic appetite. Consumers endowed with a weak intrinsic appetite and high taste for junk food might not be concerned with overweightness and obesity. Due to their overall
low intake of food they might allow themselves a diet rich in the tastier junk food. Yet a large marginal adverse effect of the physiologically inadequate diet on health moderates the consumption of junk food and, in turn, improves the rational consumer’s stationary health.

If the rate of time preferences is lower than the intrinsic endurance-appetite ratio (i.e., \( \rho < \eta / \beta \)), the steady state, as displayed by figure 2, is unique, interior and the stationary health and productivity level is given by:

\[
x_{ss} = 0.5 \left\{ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right\} + \sqrt{1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}^2} + \frac{(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}. \tag{16}
\]

If \( \eta / \beta < \rho < \eta / \beta + \frac{(1 + \eta / \beta)}{\alpha - p} \left[ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right]^2 \) there are, as displayed by Figure 3, two possible interior steady states: one with low level of health and the other with high level of health.

The construction of the phase-plane diagrams in Appendix B and the analysis of the steady-states’ nature in Appendix C reveal that, as long as the consumer’s rate of time preference is lower than a critical level (\( \theta \) in Eq. (C10)), the consumer’s health and junk-food consumption oscillate around the interior steady states. The explanation of these oscillations is as follows. An increase in junk-food consumption deteriorates health and, in turn, productivity. The decline in health increases the value of health for the consumer and, in turn, discourages junk-food consumption. At the same time, the decrease in income further contributes to a decline in the consumption of junk food by a consumer for whom this commodity is a normal good. Contrarily, the decrease in income encourages junk-food consumption by a consumer for whom it is an inferior good. However, the health-value-appreciation effect is dominant and the net change in junk-food consumption is negative, leading to a health and productivity improvement.
and, in turn, to a decline in the value of health and to a rise in income. Subsequently, junk-food consumption is increased, and so forth.

Appendix C also suggests that if the current consumption of junk food accelerates the consumption of junk food \( \left( \frac{\partial c_j}{\partial c}(x_{ss}, c_{jss}) > 0 \right) \), the magnitude of the fluctuations of health and junk-food consumption increases with the passage of time (i.e., the steady state is an asymptotically unstable spiral). The acceleration of the ensuing junk-food consumption by the current junk-food consumption might represent a case of a consumer who is addicted to junk food. In which case, the acceleration is explained by a rise in the consumer’s tolerance to some addictive substances contained in junk food. If, in contrast, a repulsion (e.g. nausea) is caused by the large quantities of sugar, fat and salt usually contained in junk food, the current consumption of junk food decelerates the ensuing consumption of junk food \( \left( \frac{\partial c_j}{\partial c}(x_{ss}, c_{jss}) < 0 \right) \), and the oscillations of health and junk-food consumption are dampened over time (i.e., the steady state is an asymptotically stable spiral). If the change in the consumption of junk food is not affected by its current consumption \( \left( \frac{\partial c_j}{\partial c}(x_{ss}, c_{jss}) = 0 \right) \), health and junk-food consumption display a fixed cyclical pattern (i.e., the steady state is a center).

7. Concluding remarks

The physiologically optimal diet, which excludes junk food, is not necessarily the rational one. As long as the consumers’ relative taste of junk food exceeds the relative price of junk food, some consumption of junk food is rational despite its adverse effect on the consumers’ health, productivity and longevity. The higher the consumer’s intrinsic endurance, the greater the consumer’s complacency and, in turn, junk-food consumption. The steady-state combinations of junk-food consumption and
health of rational consumers are not necessarily stable. Explosive oscillations of junk-food consumption and health are possible, especially in the case of addiction to some ingredients contained in junk food, and lead to intensified fluctuations in consumers’ productivity.

Governments can increase the consumers’ and the national levels of health and output and reduce their volatility by taxing junk-food consumption. Consider an economy with \(N\) consumers who maximize their expected lifetime utility. These consumers’ full-capacity incomes are equal to their physiologically optimal diets: 
\[ \dot{y}_1 = c_{h1}^0, \dot{y}_2 = c_{h2}^0, \dot{y}_3 = c_{h3}^0, \ldots, \dot{y}_N = c_{hN}^0. \] 
Their initial health conditions are 
\[ x_1(0), x_2(0), x_3(0), \ldots, x_N(0). \] 
Their relative tastes are \(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_N\). Their (full) prices of junk food, relative to healthy food, are \(p_1, p_2, p_3, \ldots, p_N\) and may differ from one another in accordance with their local market conditions and personal food preparation ability and value of time. Suppose that the junk food’s relative price is lower than its relative taste for some, or all, of the consumers and hence stimulates junk-food consumption. A tax rate that bridges the largest positive gap between the relative price and the relative taste of junk food (i.e., 
\[ \max\{\alpha_i - p_i, \alpha_2 - p_2, \alpha_3 - p_3, \ldots, \alpha_N - p_N\} \]) ensures the choice of a junk-free diet by every member of the society. Noting that the rate of change of the \(i\)-th consumer’s health and income is given by 
\[ 1 - \delta[c_{ji}(t)/c_{hi}^0][1-x_i(t)], \] 
the application of this tax rate on junk-food consumption maximizes the aggregate health improvement and facilitates the convergence of the actual aggregate income from 
\[ \sum_{i=1}^{N} x_i(0)\dot{y}_i \] 
to the aggregate potential income 
\[ \sum_{i=1}^{N} \dot{y}_i \] 
at the highest rate, 
\[ 1 - \sum_{i=1}^{N} x_i(t)^2 \dot{y}_i / \sum_{i=1}^{N} x_i(t)\dot{y}_i. \] 
(See Appendix D.) Although information about the
consumers’ prices and tastes is not available and costly, it is possible to obtain maximum and fastest growth by setting the tax rate on junk food at a very high level.

The case for taxing junk food can be further supported by other aspects, which are not included in the analysis, such as negative externalities and imperfect information. Junk-food consumption reduces health and hence increases the demand for health care. Greater health-care costs are paid for through higher fees and/or taxes. Hence, one’s junk-food consumption adversely affects other people’s welfare. It is also possible that the consumers are not well informed about the health costs of eating junk food and that the costs of acquiring this information are too high. However, a tax-induced universal abstinence is not Pareto-superior to the free-market outcome that includes cases of partial and total indulgence. People who have a high taste for, and low (full) price of, junk food might be especially hurt by a tax on junk food. Furthermore, taxing junk food might not achieve the intended objective of increasing productivity if for many consumers junk food is a Giffen good. The tax would reduce their real income, increase their consumption of junk food and, in turn, deteriorate their health and lower their productivity.
Appendix A: The solution of the optimal-control problem and steady states

\[ H(t) = \Phi(x)e^{-\rho \xi}[(\alpha - p)c_j + xe^{\alpha \xi}]^\beta \lambda + \lambda[1 - \delta(c_j / c_h)]\frac{1}{1 - \delta(c_j / c_h)} \]  

(A1)

\[ \dot{\lambda} = -\frac{\partial H}{\partial x} = -[\Phi Z \beta Z^{\beta - 1} e^{-\rho \xi} - \lambda(1 - 2x)[1 - \delta(c_j / c_h)]] \]  

(A2)

\[ \frac{\partial H}{\partial c_j} = \Phi e^{-\rho \xi} Z \beta Z^{\beta - 1} e^{-\rho \xi} - \lambda(1 - \delta(c_j / c_h))x(1 - x) = 0 \]  

(A3)

Eq. (14) is obtained as follows. By differentiating the optimality condition (A3) with respect to time, substituting the right-hand sides of the adjoint equation (A2) and the optimality condition (A3) for \( \dot{\lambda} \) and \( \lambda \):

\[-\Phi e^{-\rho \xi} Z \beta Z^{\beta - 1} (1 - \beta)(\alpha - p) - (1 - \beta) \Phi e^{-\rho \xi} Z \beta Z^{\beta - 2} (\alpha - p)(\alpha - p)\dot{c}_j + c_h \dot{x}] + \Phi e^{-\rho \xi} Z \beta Z^{\beta - 1} (\alpha - p)

(A4)

\[ + \left( [\Phi Z \beta Z^{\beta - 1} e^{-\rho \xi} + (1 - 2x)[1 - \delta(c_j / c_h)] \Phi e^{-\rho \xi} Z \beta Z^{\beta - 1} (\alpha - p)] \right) \frac{(1 - x)(\delta / c_h)}{(1 - x)(\delta / c_h)} x \]  

Multiplying both sides by \( e^\delta / \Phi Z \beta Z^{\beta - 2} \Phi Z \beta Z^{\beta - 2} (1 - \beta)(\alpha - p) \) and collecting terms:

\[ \left( \Phi - \rho \right) Z - (1 - \beta)[(\alpha - p)\dot{c}_j + c_h \dot{x}] + \left( \Phi \frac{Z^2}{\beta(\alpha - p)} + \frac{Zc_h}{(\alpha - p)} \right)(1 - x)(\delta / c_h) x \]  

(A5)

\[ + (1 - 2x)[1 - \delta(c_j / c_h)] Z - \left( \frac{(1 - 2x)Z}{(1 - x)} \right) Z \dot{x} = 0 \]

Recalling that \( \Phi / \Phi = \eta(x / x) \) and \( \Phi_x / \Phi = \eta / x \).
By rearranging terms,

\[
\begin{align*}
-\rho Z - (1 - \beta)(\alpha - p)\dot{c}_j + \left(\frac{\eta}{\beta x} Z + c^0_h\right)(1 - x)(\delta / c^0_h)x \frac{1}{(\alpha - p)Z} Z \\
+ (1 - 2x)[1 - \delta(c_j / c^0_h)]Z = \left\{\frac{(1 - 2x)}{(1 - x)x} - \frac{\eta}{x}\right\}Z + (1 - \beta)c^0_h \frac{\dot{x}}{x} = 0
\end{align*}
\]  

(A7)

Subsequently,

\[
\begin{align*}
\dot{c}_j = -\frac{\rho}{(1 - \beta)(\alpha - p)} Z + \left(\frac{\eta}{\beta x} Z + c^0_h\right)(1 - x)(\delta / c^0_h)x \frac{1}{(\alpha - p)Z} Z \\
+ \frac{1}{(1 - \beta)(\alpha - p)}(1 - 2x)[1 - \delta(c_j / c^0_h)]Z \\
- \frac{1}{(1 - \beta)(\alpha - p)}\left\{\left\{\frac{(1 - 2x)}{(1 - x)x} - \frac{\eta}{x}\right\}Z + (1 - \beta)c^0_h \frac{\dot{x}}{x}\right\}
\end{align*}
\]  

(A8)

or, equivalently,

\[
\begin{align*}
\dot{c}_j = -\frac{Z}{(1 - \beta)(\alpha - p)} \left\{\rho - (1 - 2x)[1 - \delta(c_j / c^0_h)] \\
- \frac{(\eta Z + \dot{\gamma})(1 - x)(\delta / c^0_h)x}{(\alpha - p)} \\
+ \left\{\frac{(1 - 2x)}{(1 - x)x} - \frac{\eta}{x}\right\} + \frac{(1 - \beta)c^0_h}{Z}\right\} \dot{x}
\right\}
\]  

(A9)

Recalling that \(\frac{\dot{x}}{(1 - x)x} = 1 - \delta(c_j / c^0_h)\) and \(Z = (\alpha - p)c_j + xc^0_h\),

\[
\dot{c}_j = \left\{\frac{(\alpha - p)c_j + xe^0_h}{(1 - \beta)(\alpha - p)}\right\} \left\{\rho - \frac{(\eta Z + c^0_h)(1 - x)(\delta / c^0_h)x}{(\alpha - p)} - \left\{\frac{\eta}{x} - \frac{(1 - \beta)c^0_h}{Z}\right\}\dot{x}\right\}.
\]  

(A10)

Eq. (15) is obtained as follows. The substitution of \(\dot{c}_j = \dot{x} = 0\) into (A6) implies:
\[-pZ + \left( \frac{\eta}{x}Z^2 \right) + \frac{Zc_h^0}{\beta(\alpha - p) + (\alpha - p)}(1 - x)(\delta/c_h^0)x + (1 - 2x)[1 - \delta(c_j/c_h^h)]Z = 0. \quad (A11)\]

By rearranging terms,
\[-\beta(\alpha - p)\rho + (\delta/c_h^0)\eta Z(1 - x) + (\delta/c_h^0)\beta(1 - x)x
+ \beta(\alpha - p)(1 - 2x)[1 - (\delta/c_h^0)c_j] = 0 \quad (A12)\]

Recalling that $c_{j_{ss}} = c_h^0 / \delta$,
\[-\beta(\alpha - p)\rho + (\delta/c_h^0)\eta[(\alpha - p)/(\delta/c_h^0)] + xc_h^0(1 - x)
+ (\delta/c_h^0)\beta c_h(1 - x)x = 0 \quad (A13)\]

Rearranging terms,
\[-\beta(\alpha - p)\rho + [(\alpha - p)\eta + \delta\eta x](1 - x) + \delta(1 - x)x = 0 \quad (A14)\]

or, equivalently,
\[(\beta + \eta)\delta x^2 - [(\beta + \eta)\delta - (\alpha - p)\eta]x - (\alpha - p)(\eta - \beta \rho) = 0 \quad (A15)\]

or, equivalently,
\[x_{ss}^2 - \frac{[(\beta + \eta)\delta - (\alpha - p)\eta]}{(\beta + \eta)\delta}x_{ss} - \frac{(\alpha - p)(\eta - \beta \rho)}{(\beta + \eta)\delta} = 0 \quad (A16)\]

or, equivalently,
\[x_{ss}^2 - \left(1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right)x_{ss} - \frac{(\alpha - p)(\eta - \beta \rho)}{(\eta + \beta)\delta} = 0 \quad (A17)\]

Consequently, the individual’s steady-state health level(s) is (are) given by Eq. (15)
Appendix B: Phase-plane diagrams

From (6), the isocline $\dot{x} = 0$ is given by a horizontal line in the plane spanned by $x$ and $c_j$:

$$c_j = c_h^0 / \delta .$$  

(B1)

From (14) and the definition of $Z$, along the isocline $\dot{c}_j = 0$

$$\frac{\eta}{(\alpha - p)} [xc_j + x_c^0(1-x)(\delta / c_h^0)x] = 0$$

(B2)

By rearranging terms the isocline $\dot{c}_j = 0$ is given by:

$$c_j = \left[ \frac{\rho \beta / (\delta / c_h^0) \eta}{(1-x)^2} - \frac{c_h^0[1 + \beta / \eta]}{(\alpha - p)} \right] x$$

(B3)

The slope of the isocline $\dot{c}_j = 0$ is:

$$\frac{dc_j}{dx} \bigg|_{\dot{c}_j = 0} = (\rho \beta / \tilde{\delta} \eta) \frac{1}{(1-x)^2} - \frac{c_h^0[1 + \beta / \eta]}{(\alpha - p)} < \frac{\eta \beta}{\tilde{\delta} \eta} (\alpha - p) = c_h^0[1 + \beta / \eta](1-x)^2$$

(B4)

where $\tilde{\delta} \equiv \delta / c_h^0$.

Recalling that $0 \leq x \leq 1$ and $\alpha - p > 0$, $\frac{dc_j}{dx} \bigg|_{\dot{c}_j = 0} > 0$ as $(1-x)^2 < \frac{\rho \beta / \tilde{\delta} \eta (\alpha - p)}{c_h^0[1 + \beta / \eta]}$

and $\frac{d^2c_j}{dx^2} \bigg|_{\dot{c}_j = 0} > 0$. The isocline $\dot{c}_j = 0$ is U-shaped in the plane spanned by $x$ and $c_j$. 

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In the case of \( \rho = \eta / \beta \), the second term in the discriminant is equal to zero and hence \( x_{ss} = 0 \) or \( x_{ss} = 1 - [(\alpha - p)\eta / (\eta + \beta)\delta] \). Note further that in this case

\[
    c_{j, j=0} = \frac{\rho}{\eta / \beta} (c^0_h / \delta) = (c^0_h / \delta) \text{ when } x = 0.
\]

In the case of \( \rho < \eta / \beta \),

\[
    \left( 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right) < \sqrt{\left( 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right)^2 + \frac{4(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}}.
\]

Hence, \( x_{ss2} < 0 \) and only \( 0 < x_{ss1} < 1 \). Note further that for \( x = 0 \)

\[
    c_{j, j=0} = \frac{\rho}{\eta / \beta} (c^0_h / \delta) < (c^0_h / \delta) \text{ since } \rho < \eta / \beta \). The isocline \( \dot{c}_j = 0 \) intersects the isocline \( \dot{x} = 0 \) only once as displayed by Figure 2.

In the case of \( \eta / \beta < \rho < \eta / \beta + \frac{(1 + \eta / \beta)\delta}{\alpha - p} \left[ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right]^2 \), the discriminant in Eq. (15) is positive,

\[
    \left( 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right) > \sqrt{\left( 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right)^2 + \frac{4(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}}
\]

and \( c_{j, j=0} = \frac{\rho}{\eta / \beta} (c^0_h / \delta) > (c^0_h / \delta) \) for \( x = 0 \). In this case, the isocline \( \dot{c}_j = 0 \) may intersect the isocline \( \dot{x} = 0 \) twice as displayed by Figure 3.

Comment: The discriminant in Eq. (15) is zero, or negative, when \( \rho \) is equal to, or greater than, \( \eta / \beta + \frac{(1 + \eta / \beta)\delta}{\alpha - p} \left[ 1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta} \right]^2 \).
Appendix C: The properties of the steady states

To assess the steady states’ properties consider the state-transition matrix \( \Omega \) of the linearized form of Eq. (6) and Eq. (14) in the vicinity of steady state:

\[
\Omega = \begin{bmatrix}
\frac{\partial \hat{x}}{\partial x}(x_{ss},c_{j_{ss}}) & \frac{\partial \hat{x}}{\partial c_j}(x_{ss},c_{j_{ss}}) \\
\frac{\partial \hat{c}}{\partial x}(x_{ss},c_{j_{ss}}) & \frac{\partial \hat{c}}{\partial c_j}(x_{ss},c_{j_{ss}})
\end{bmatrix}.
\] (C1)

Since

\[
\frac{\partial \hat{x}}{\partial c_j}(x_{ss},c_{j_{ss}}) = (1 - \tilde{c}_{j_{ss}})(1 - 2x_{ss}) = (1 - \tilde{\delta}(1/\tilde{\delta}))(1 - 2x_{ss}) = 0
\] (C2)

the eigenvalues of \( \Omega \) are given by

\[
\psi_{1,2} = 0.5\left(\frac{\partial \hat{c}}{\partial c_j}(x_{ss},c_{j_{ss}}) \pm \sqrt{\left[\frac{\partial \hat{c}}{\partial c_j}(x_{ss},c_{j_{ss}})\right]^2 + 4\frac{\partial \hat{c}}{\partial x}(x_{ss},c_{j_{ss}})\frac{\partial \hat{x}}{\partial c_j}(x_{ss},c_{j_{ss}})}\right)
\] (C3)

with:

\[
\frac{\partial \hat{x}}{\partial c_j}(x_{ss},c_{j_{ss}}) = -\tilde{\delta}(1 - x)x < 0
\] (C4)

\[
\frac{\partial \hat{c}}{\partial c_j}(x_{ss},c_{j_{ss}}) = -\frac{1}{(1 - \beta)}(\rho - R_{2ss}) + R_{1ss}(\eta / \beta)\tilde{\delta}(1 - x_{ss})
\] (C5)

\[
+ R_{1ss} R_{3ss} \tilde{\delta}(1 - x_{ss}) x_{ss}
\]

\[
\frac{\partial \hat{c}}{\partial x}(x_{ss},c_{j_{ss}}) = -\frac{c_k^o}{(1 - \beta)(\alpha - p)}(\rho - R_{2ss})
\] (C6)

and where,

\[
R_{1ss} = \frac{(\alpha - p)/\tilde{\delta} + x_{ss}c_k^o}{(1 - \beta)(\alpha - p)} > 0
\] (C7)
\[
R_{zs} = \frac{\eta - Z_{ss} + c_h^0 (1 - x_{ss}) \delta_{ss}}{(\alpha - p)} = \left\{ \frac{\eta + \frac{[(\eta / \beta) + 1] \delta}{(1 - \beta) x_{ss}}}{(\alpha - p)} \right\} (1 - x_{ss}) \geq 0 \text{ as } x_{ss} \geq 0
\]  
(C8)

\[
R_{zs} = \left[ \frac{\eta}{x_{ss}} - \frac{(1 - \beta) c_h^0}{Z_{ss}} \right].
\]  
(C9)

From Eq. (C6), \( \frac{\partial c_j}{\partial x} (x_{ss}, c_{j ss}) > 0 \) for consumers endowed with

\[
\rho < \theta = R_{zs} + R_{1ss} \left\{ \frac{[(\eta / \beta) / (\delta_{ss})] + [(\eta / \beta) + 1] c_h^0 \delta (1 - 2x_{ss}) + \left( p c_h^o / \beta \right) (1 - x_{ss}) (1 - \beta)}{c_h^0} \right\} > 0.
\]  
(C10)

In the vicinity of \( x_{ss} = 0 \), \( \frac{\partial c_j}{\partial x} (x_{ss}, c_{j ss}) > 0 \) since

\[
\theta_{x_{ss}=0} = \frac{(\alpha - p) / \delta}{(1 - \beta)(\alpha - p)} \left\{ \frac{[(\eta / \beta) / 0] + [(\eta / \beta) + 1] c_h^0 \delta + \left( p c_h^o / \beta \right) (1 - \beta)}{c_h^0} \right\} \rightarrow \infty.
\]

This explains the direction of the vertical arrows at the vicinity of \( x_{ss} = 0 \) in Figure 1. The vertical arrows in the vicinity of the interior steady states in the phase-plane diagrams are displayed under the assumption that \( \rho < \theta \). The directions of the horizontal arrows in the phase-plane diagrams are explained by Eq. (C4). The directions of the horizontal and vertical arrows indicate that the steady states in the phase-plane diagrams are centers if \( tr \Omega = \frac{\partial c_j}{\partial c_j} (x_{ss}, c_{j ss}) = 0 \), asymptotically stable spirals if \( tr \Omega = \frac{\partial c_j}{\partial c_j} (x_{ss}, c_{j ss}) < 0 \), or asymptotically unstable spirals if

\[
tr \Omega = \frac{\partial c_j}{\partial c_j} (x_{ss}, c_{j ss}) > 0. \text{ In the case of } x_{ss} = 0, \frac{\partial c_j}{\partial c_j} = \frac{1}{(1 - \beta)} [(\eta / \beta) - \rho] = 0 \text{ and}
\]
hence the associated steady state in Figure 1 is a centre. In the cases of the interior steady states \( x_{ss} > 0 \) the sign of \( tr\Omega \) is not clear. (See Eq. (C5).)

**Appendix D: Health and productivity maximizing tax**

Recalling Eq. (2),

\[
\hat{y}_i(t) \quad \frac{\dot{y}_i(t)}{y_i(t)} = \frac{\dot{x}_i(t)}{x_i(t)} = \frac{\ddot{x}_i(t)}{x_i(t)} = \{1 - \delta [c_{ji}(t)/c_{hi}^\alpha] \} \{1 - x_i(t)\}.
\]

(D1)

Recalling that \( p > \alpha \) leads to abstinence, an immediately implemented tax rate on junk food that is equal to \( \max\{(\alpha_1 - p_1), (\alpha_2 - p_2), (\alpha_3 - p_3), \ldots, (\alpha_N - p_N)\} \) ensures that every consumer immediately chooses a diet free of junk food (i.e., \( c_{hi}(t) = y_i(t) = x_i(t)\hat{y}_i \forall i = 1,2,3,\ldots,N \)). Recalling Eq. (6) and Eq. (2) and that \( \hat{y}_i = c_{hi}^\alpha \), the health-growth rate is, in turn, maximal and the convergence of the actual aggregate product, \( Y(t) = \sum_{i=1}^{N} x_i(t)\hat{y}_i \), to the potential aggregate product, \( \sum_{i=1}^{N} \hat{y}_i \), is feasible and most rapid:

\[
\frac{\dot{Y}(t)}{Y(t)} = \frac{\sum_{i=1}^{N} \ddot{y}_i(t)}{\dot{y}_i(t)} = \frac{\sum_{i=1}^{N} \dot{x}_i(t)\hat{y}_i}{\dot{x}_i(t)\hat{y}_i} = \frac{\sum_{i=1}^{N} x_i(t)[1 - x_i(t)]\hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = 1 - \frac{\sum_{i=1}^{N} x_i(t)^2 \ddot{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i}.
\]

(D2)
Figure 1. Phase-plane diagram with $\rho = \eta / \beta$
Figure 2. Phase-plane diagram with $\rho < \eta / \beta$
Figure 3. Phase-plane diagram with $\eta / \beta < \rho < \eta / \beta + \frac{(1 + \eta / \beta) \delta}{\alpha - p} \left[1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right]^2$.
References


