This paper analyzes the relationships between the values and dispersion of residential properties and the environmental-health quality of their locations. It constructs residents’ health-adjusted lifetime-utility function by combining satisfaction from consumption over the lifespan with risk to life from living in an environmentally unhealthy location. It employs this utility function to analyze willingness to pay for environmental-health quality, choice of location and residential dispersion and its relationship with income distribution. (JEL I10, I12, R20, D91)

Keywords: Environmental health; health risk; life expectancy; lifetime utility; property value; residential location; residential dispersion.

1. Introduction

The environmental quality of a place of residence has strong long-term implications for the health of the decision-makers and their dependents. The hypothesis that certain facilities and land use constitute environmental health hazard and hence deter demand for adjacent residential properties, has been extensively tested. Applying hedonic pricing methods and other techniques to cross-section and time-series data, a large number of empirical studies (including the seminal articles by Harrison and Rubinfeld, 1978; and Freeman, 1979; and the more recent papers by Michaels and Smith, 1990; Kohlhase, 1991; Kiel and McClain, 1995a, 1995b; and McCluskey and Rausser, 2003a, 2003b) have lent support to a positive correlation between residential-property prices and distance from sources of environmental health hazards and, consequently, to the assertion that the values of residential properties reflect people’s concerns with the environmental quality of their location. These concerns arise mainly from worries about health risk and stigma (McClelland, Schulze and Hurd, 1990, Wandersman and Hallman, 1993; Lober and Green, 1994).

The objective of this paper is to provide a formal derivation of the effects of sited sources of environmental health hazards on the values, location and dispersion of residential properties. The empirical benefit from this formal derivation is the identification of factors that are missing in hedonic regression equations of willingness to pay for an environmentally cleaner neighbourhood. The theoretical analysis is based on the assumptions that the duration of life is uncertain and depends upon the environmental quality of the place of residence; that the quality of a neighborhood is eroded by health

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1 The case of discovered environmental hazards is beyond the scope of the present paper. This case requires a large modification of the model to incorporate another stochastic feature – uncertainty about the existence of an environmental hazard and its location.
hazards posed by nearby facilities and land use; and that residents’ lifetime utility from consumption increases with the probability of survival associated with the environmental health quality of their neighborhood and the period of exposure to hazardous facilities and land use. Following a construction of residents’ health-adjusted utility function, the paper analyses residents’ willingness to pay for environmental quality, their choice of location and their aggregate level of residential dispersion and its relationship with income distribution.

The analysis emphasizes the roles of consumption-elasticity of utility and sensitivities of personal health and income and market rent to environmental hazard in the determination of residents’ willingness to pay for environmental quality and in the determination of residential location and dispersion. Ignoring these personal sensitivities and consumption elasticity renders the estimates of the determinants of willingness to pay for an environmentally cleaner residential neighbourhood obtained with cross-section and panel data biased. In this respect, the analysis highlights the importance of measuring the sensitivities of the individuals’ health and earning to neighbourhood’s environmental quality as well as the consumption elasticity of their utility. It advocates the incorporation of these measurements into the database, and the expansion of the hedonic price regression equation accordingly, for improving the estimation of the determinants of willingness to pay for living in an environmentally cleaner location. Furthermore, the consideration of the said personal sensitivities and consumption elasticity provides an explanation to observed corner solutions - living in the neighbourhood with the highest, or lowest, environmental quality - which cannot be entirely explained by income, as implicitly suggested by the conventional hedonic regression models.

The analysis is organized as follows. Section 2 constructs a resident information about the environmental health quality of every location is perfect information about the environmental health quality of every location is perfect’s health-adjusted lifetime utility function by combining the resident’s satisfaction from consumption over the lifespan with the risk to life from residing in a hazardous environment. Section 3 employs this lifetime utility function to analyze the resident’s willingness to pay for environmental quality. Section 4 analyses the resident’s choice of location. Section 5 extends the analysis of the choice of location to the case where the resident’s productivity is affected by the environmental quality of his neighborhood. Section 6 analyzes the residential dispersion of a heterogeneous population and its relationship with income distribution. Section 7 summarizes the main results and indicates the possible relationship between income disparity, political power and the location and persistence of hazardous facilities and land use.

2. Resident’s health-adjusted utility

It is possible that on major and non-easily reversible choices, such as place of residence, decisions are made in accordance with expected-lifetime-utility maximization. Consider a lifetime-utility maximizer, J, whose rate of time preference, $\rho$, is time invariant and positive, but not very large, revealing that he cares about his future utilities from consumption. Being farsighted, J’s planning horizon is long – infinite, for

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2 Due to large financial, psychological and social costs.
tractability. However, J is aware of the uncertainty about his existence and of the effect of an environmental health hazard near his place of residence on his probability of survival. The instantaneous income of J is $y_{jt}$. His instantaneous residential rent (user cost in the case of ownership) is $R_t$. His instantaneous spending on consumption is $y_{jt} - R_t$. His instantaneous utility from consumption is $u_j(y_{jt} - R_t)$, displaying $u'_{jt} > 0$ and $u''_{jt} < 0$. Being an expected-lifetime-utility maximizer, J multiplies his accumulated utility between the starting point of his planning horizon $0$ to his possible time of death $t$, $\int_0^t e^{-\rho_j \tau} u_j(y_{jt} - R_t) d\tau$, by his probability of dying at time $t$, $f_{jt}$. The sum of all the products of $f_{jt}$ and $\int_0^t e^{-\rho_j \tau} u_j(y_{jt} - R_t) d\tau$ associated with any possible time of death $0 \leq t \leq \infty$ is his expected lifetime-utility:

$$V_j = \int_0^\infty f_{jt} \int_0^t e^{-\rho_j \tau} u_j(y_{jt} - R_t) d\tau dt .$$  \hspace{1cm} (1)

Integrating by parts, J’s expected lifetime-satisfaction can be rendered as the sum of his discounted instantaneous utility from consumption accruing during his planning horizon and weighted by his probability of prevailing:

$$V_j = \int_0^\infty [1 - F_{jt}] e^{-\rho_j t} u_j(y_{jt} - R_t) dt$$  \hspace{1cm} (2)

where, $F_{jt}$ denotes the cumulative density function associated with $f_{jt}$ and indicates J’s probability of dying by $t$. Hence, $1 - F_{jt}$ displays J’s probability of living beyond $t$. (See Appendix A.)

Spending much of his time at his home and neighborhood, J’s probability of dying by $t$ depends upon the health quality of his neighborhood’s environment during the period $(0,t)$ and upon the sensitivity of his health to this quality. J takes his neighborhood’s environmental health quality to be eroded by hazards posed by nearby facilities and undesired land use, which he expects to persist indefinitely. He therefore believes that the environmental health quality of any location affected by these facilities and land use is time-invariant. J also believes that the rent and his income in any location are time-invariant. Hence, he intends to stay in one location after making his choice. In other words, J expects to be exposed to the initial environmental health quality, $\theta$, of his chosen neighborhood for the rest of his life. He takes $\theta$ to belong to the unit interval $(0,1)$, where 1 represents the least hazardous environment and 0 the most hazardous environment. The sensitivity of J’s health to the health quality of his residential environment is a non-negative scalar $\alpha_j$ $(0 \leq \alpha_j \leq 1)$. J’s probability of dying by $t$

$^3$ The assumption of infinite planning horizon can be further justified by considering J to be a representative of a household and/or a family.

$^4$ There is no saving, or there is no distinction between saving and consumption, for simplicity.

$^5$ Consistently with the aforementioned empirical studies, $\theta$ may rise with the distance from the source(s) of the environmental hazard.
declines with $\theta$ and $\alpha_j$, but rises with the period of exposure $(0,t)$ to the hazardous facilities and land use. Namely, $F_{jt} = F_j(\theta, \alpha_j, t)$ with $\partial F_j / \partial \theta < 0$, $\partial F_j / \partial \alpha < 0$ and $\partial F_j / \partial t > 0$.

For convenience, the following explicit form is used to approximate $J$’s probability of dying by $t$:

$$F_{jt} = 1 - e^{-\mu_j t} \theta^{\alpha_j}.$$  

(3)

In view of this specification, $\alpha_j$ can be interpreted as $J$’s elasticity of health with respect to the environmental health quality of his neighborhood and $e^{-\mu_j t}$ as the effect of the duration of $J$’s exposure to hazardous facilities and land use on his probability of survival. Since $\mu_j = -[d(1 - F_j)/dt]/[1 - F_j]$, $\mu_j$ is a positive scalar indicating the rate of decline of $J$’s survival probability due to his continued exposure to hazardous facilities and land use.

By substituting Eq. (3) into Eq. (2) and the assumption of time-invariant income and rent,

$$V_j = \int_0^\infty e^{-(\rho_j + \mu_j) t} \theta^{\alpha_j} u_j(y_j - R) dt,$$  

(4)

which can now be interpreted as $J$’s lifetime sum of instantaneous utilities from consumption adjusted to the environmental health quality of his location and discounted by his time preference and the effect of his continued exposure to hazardous facilities and land use. By integrating the right-hand side of Eq. (4), $J$’s expected lifetime utility is equal to the ratio of his health-adjusted instantaneous utility from consumption and his full discounting rate:

$$V_j = \theta^{\alpha_j} u_j(y_j - R) / (\rho_j + \mu_j).$$  

(5)

The interpretation of this expected lifetime-utility function can be broadened to include the effect of stigma. If the public image of a neighborhood worries $J$ and is correlated with the environmental quality of the neighbourhood, $\alpha_j$ is the sum of $J$’s health sensitivity to spatial health quality and $J$’s sensitivity to the public image of his place of residence.

3. Residents’ willingness to pay for environmental health

Suppose that location A is less affected than location B by health impeding hazardous facilities and land use and is accurately perceived as such, i.e., $\theta_A > \theta_B$. Recalling Eq. (5) and assuming that income is not affected by the environmental health of the residential neighborhood, $J$ is indifferent between location A and location B as long as his full rents$^7$, $R_A^j$ and $R_B^j$, on structurally identical residential properties in these locations satisfy:

$$\theta_A^{\alpha_j} u_j(y_j - R_A^j) / (\rho_j + \mu_j) = \theta_B^{\alpha_j} u_j(y_j - R_B^j) / (\rho_j + \mu_j).$$  

(6)

$^6$ This assumption is relaxed in section 5.

$^7$ Including traveling costs to places of business and social activities.
In other words, J is indifferent between the two locations if there is equality between the ratio of his instantaneous utilities from consumption and the inverse of the corresponding ratio of the impacts of the environmental health qualities of A and B on his well-being:

\[
\frac{u_J(y_j - R^j_A)}{u_J(y_j - R^j_B)} = (\theta_B / \theta_A)^{\alpha_j}.
\] (7)

**Corollary 1:** J is willing to pay a higher full rent for a residential property in an environmentally less hazardous location if his health is sensitive to the environmental health quality of his place of residence.\(^8\) (See proof in Appendix B.)

When \(\alpha_j\) is taken to be the sum of J’s health sensitivity to environmental health quality and J’s concern about his neighborhood’s public image, this corollary encompasses sensitivity to stigma.

J’s willingness to pay for the environmental health quality difference between location A and location B is further explored by using the analytically convenient isoelastic function of instantaneous utility from consumption

\[
u_{ji} = (y_j - R^j_i)^{\beta_j}, \quad 0 < \beta_j < 1,
\] (8)

for any location \(i = A, B\). By substituting this explicit form into Eq. (7), J’s willingness to pay extra rent for the environmental-health-quality difference between location A and location B is:

\[
R^j_A - R^j_B = [1 - (\theta_A / \theta_B)^{-\alpha_j / \beta_j}](y_j - R^j_B).
\] (9)

By taking the duration of residential properties to be very long, infinite for tractability, and the market capitalization rate of an ordinary residential property to be equal to the full discounting rate \((\rho_j + \mu_j)\) of a perfectly rational person such as J and summing the discounted willingness to pay extra rent over an infinite period, the difference between J’s highest bid on an ordinary residential property in location A \((P^j_A)\) and J’s highest bid on an identical property in location B \((P^j_B)\) can be expressed as:

\[
P^j_A - P^j_B = [1 - (\theta_A / \theta_B)^{-\alpha_j / \beta_j}](y_j - R^j_B) / (\rho_j + \mu_j).
\] (10)

**Corollary 2:** The positive effect of the A-B environmental-health-quality ratio on J’s A-B highest-bid differential on structurally identical residential properties is intensified by J’s income and health sensitivity, but is moderated by J’s consumption elasticity of utility, rate of time preference and rate of decline of survival probability stemming from continued exposure to hazardous facilities and land use. (See proof in Appendix B.)

4. **Optimal location of residence**

\(^8\) This corollary is consistent with Smith and Desvousges’ (1986) finding that respondents are willing to pay between $2472 and $3199 more for residential properties located a mile further from a hazardous waste landfill.
Suppose that source of the environmental health hazard is sited (e.g., incinerator) and sufficiently long established, so that information about the environmental health quality of the surrounding neighbourhoods is already efficiently conveyed by the market rents and locations are continuously ranked by their environmental health quality $\theta$ within the unit interval (0,1). Consistent with the previous section’s findings about willingness to pay rent on structurally identical residential properties in locations endowed with different environmental health qualities, the market rents of ordinary residential properties rise with the environmental health quality of their location and can be approximated by a differentiable function $R(\theta)$ with $R'(\theta) > 0$. By substituting this rent function into Eq. (5),

$$\theta_j^* = \arg\max \{\theta^\alpha_j u_j(y_j - R(\theta_j))/\rho_j + \mu_j\} = \frac{\alpha_j u_j(y_j - R(\theta_j^*))}{R'(\theta_j^*) u'(y_j - R(\theta_j^*))}$$

(11)

if $\frac{y_j - R(\theta_j^*)}{R'(\theta_j^*)} > 1 - \frac{\beta_j}{1 + \alpha} \theta_j^*$ (see Appendix C). Due to the assumption of time-invariant relative environmental health qualities, $\theta_j^*$ is independent of the full discounting rate.

Suppose, for tractability, that the market rent of an ordinary residential property is linearly rising in its location’s environmental health quality from the lowest rent of $R_0$ in the most hazardous location ($\theta = 0$) to the highest rent $R_1$ in the least hazardous location ($\theta = 1$)

$$R = R_0 + (R_1 - R_0)\theta$$

(12)

where $R_1 - R_0$ can be interpreted as the market rent-gradient. Subsequently, J’s optimal residential location is where the health quality of the environment is equal to

$$\theta_j^* = \frac{\alpha_j}{\alpha_j + \beta_j} \left[ \frac{y_j - R_0}{R_1 - R_0} \right]$$

(13)

if $\frac{y_j - R_0}{R_1 - R_0} > [1 + \frac{1 - \beta_j}{1 + \alpha_j}] \theta_j^*$ (see Appendix C).

**Corollary 3:** If $\frac{y_j - R_0}{R_1 - R_0} > [1 + \frac{1 - \beta_j}{1 + \alpha_j}] \theta_j^*$, the optimal health quality of J’s place of residence is proportional to the ratio of the difference between J’s income and rent in the environmentally most hazardous location to the rent-gradient. The proportion-coefficient rises with J’s health sensitivity to environmental health quality and declines with J’s elasticity of utility from consumption. (See proof in Appendix C.)

**Corollary 4:** If $y_j \geq R_1 + (\beta_j / \alpha_j)(R_1 - R_0)$, J’s optimal place of residence is in the least health hazardous location. (See proof in Appendix C.)

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9 This assertion is compatible with the findings of Michaels and Smith (1990) Kohlhase (1991), Kiel and McClain (1995a, 1995b) and McCluskey and Rausser (2003) when $\theta$ is taken to be the distance from hazardous facilities and land use.
In other words, the least health hazardous location is chosen for residence when J’s income exceeds the rent in that location by at least the product of the rent-gradient and the ratio of J’s consumption elasticity of utility to J’s health sensitivity to environmental health quality. Of course, if \( y_j < R_0 \), J cannot afford residence even in the most health hazardous location.

5. Optimal location when environmental health quality affects productivity

It is possible that income is affected by location. If productivity is improved, and the loss of working days is reduced, by health and if health is improved by environmental health quality (i.e., \( \alpha_j > 0 \)), income rises with the health quality of the place of residence. By considering a differentiable income function \( y_j(\theta) \) with \( y_j'(\theta) > 0 \), J’s optimal residential location is now given by

\[
\theta_{j}^{**} = \arg \max \{\theta^{\alpha_j}u_j(c_j(\theta))/(\rho_j + \mu_j)\} = \frac{\alpha_ju_j(c_j(\theta_{j}^{**}))}{c'(\theta_{j}^{**})u_j'(c_j(\theta_{j}^{**}))}
\]

as long as

\[
(1 + \alpha_j)u_j'(c_j(\theta_{j}^{**})) + \theta_{j}^{**}[u_j''(c_j(\theta_{j}^{**}))c_j'(\theta_{j}^{**})] + u_j'(c_j(\theta_{j}^{**}))(c_j''(\theta_{j}^{**})) < 0,
\]

where \( c_j(\theta) \equiv y_j(\theta) - R_j(\theta) \) (see Appendix C).

Suppose, for simplicity, that income rises linearly with the location’s health quality from the lowest level of \( y_{0j} \) in the most hazardous location to the highest level \( y_{1j} \) in the least hazardous location

\[
y_j = y_{0j} + (y_{1j} - y_{0j})\theta
\]

where \( y_{1j} - y_{0j} \) can be interpreted as J’s (personal) income-gradient. By substituting Eq. (12) and Eq. (15) into Eq. (14), J’s optimal location is where the health quality is equal to

\[
\theta_{j}^{**} = \left(\frac{x_j}{\alpha + \beta_j}\right)\left(\frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})}\right)
\]

if, and only if,

\[
\frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{1j} - y_{0j})} > \frac{1}{1 + \alpha_j}\frac{1 - \beta_j}{\alpha_j}
\]

(see Appendix C).

Corollary 5 (interior solution): The optimal place of residence for J, who is sensitive to environmental health quality and facing a positive (negative) income-rent differential in the most hazardous location, is where the environmental health quality is

\[
0 < \left(\frac{x_j}{\alpha + \beta_j}\right)\left(\frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})}\right) < 1
\]

if, and only if, the effect of environmental health quality on J’s income-rent differential is negative (positive). (See proof in Appendix C.)
Corollary 6: If \( \frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{ij} - y_{0j})} > \frac{1 - \beta_j}{1 + \alpha_j} \theta_{ij}^* \), the chosen environmental health quality of J's place of residence increases with J's health sensitivity to the environmental health quality, with J's income-gradient and with J's income-market-rent differential in the most hazardous location, but decreases with J's utility's consumption elasticity and with the market rent-gradient. (See proof in Appendix C.)

Corollary 7 (corner solution): The optimal place of residence for J; whose health is sensitive to environmental health quality and who is facing a non-negative (zero) income-rent differential in the most hazardous environment and a positive (negative) difference between his income-gradient and the market rent-gradient; is in the least (most) hazardous location. (See proof in Appendix C.)

This corollary says that when people’s income in the most hazardous location is at least as large as their rent and when the environmental-health-quality effect on their income is at least as large as the environmental-health-quality effect on their rent, they maximize their consumption and minimize their risk of dying by residing in the least hazardous location. However, when their income in the most hazardous location is equal to the rent and the environmental-health-quality effect on the market rent exceeds the environmental-health-quality effect on their income, people reside in the most hazardous location since they cannot afford a safer one.

The condition for choosing the least hazardous location is more generally articulated in the following proposition.

Corollary 8: If \( y_{1j} \geq R_1 + (\beta_j / \alpha_j)((R_1 - R_0) - (y_{ij} - y_{0j})) \), J’s optimal place of residence is in the least hazardous location. (See proof in Appendix C.)

This corollary suggests that the least hazardous location is chosen for residence when the individual’s anticipated income in that location exceeds the rent in that location by at least the product of the difference between the market rent-gradient and his personal income-gradient and the ratio of his consumption elasticity of utility to his health sensitivity to environmental health quality. It further implies that the minimum anticipated income \( y_{1j}^{\text{min}} \) for residing in the least hazardous location is given by

\[
y_{1j}^{\text{min}} = R_1 + \frac{(\beta_j / \alpha_j)(y_{0j} - R_0)}{1 + (\beta_j / \alpha_j)}
\]

and, consequently,

\[
\frac{\partial y_{1j}^{\text{min}}}{\partial (\beta_j / \alpha_j)} = \frac{y_{0j} - R_0}{[1 + (\beta_j / \alpha_j)]^2} > 0 \quad \text{as} \quad y_{0j} > R_0.
\]

6. Residential dispersion and income distribution

The previous sections’ analyses of the choice of location suggest that in the absence of asymmetric information about environmental health qualities and rents and in the presence of open access, the dispersion of expected-lifetime-utility-maximizing people across locations endowed with different environmental health qualities is due to variation
in personal health sensitivity to environmental health quality, in utility’s consumption
elasticity and in income. The persistence of hazardous facilities and land use close to
residential neighborhoods might be a reflection of the affected residents’ low political
power. If political power is associated with wealth it can be expected that the more
affluent the neighborhood the more effective the lobbying against hazardous facilities and
land use in its vicinity. In other words, a strong association between residential location
and income might contribute to the persistence of undesired sources of environmental
hazards in and near low-income neighborhoods.

To facilitate the examination of the relationship between the population’s residential
dispersion and income distribution, the following analysis considers the case described in
section 4, in which personal income is not affected by the environmental health quality of
the place of residence, and assumes that there exists an interior solution to the location
choice problem of each person. The analysis initially considers the case where all people
have the same health sensitivity to environmental health quality and the same utility’s
consumption elasticity.

Let \( \alpha_j = \alpha \) and \( \beta_j = \beta \) for every person \( j = 1, 2, 3, \ldots, N \) and recall Eq. (13). Then, the
residential location of each person \( j \) is where

\[
\theta_j^* = \left( \frac{\alpha}{\alpha + \beta} \right) \left[ \frac{y_j - R_0}{R_1 - R_0} \right].
\]  

(18)

Corollary 9: When all people have identical health sensitivity to environmental health
quality and identical utility’s consumption elasticity, high (low) income earners reside in
less (more) exposed neighborhoods to hazardous facilities and land use.

Furthermore, the residential health-quality mean is

\[
E(\theta) = \left( \frac{\alpha}{\alpha + \beta} \right) \left[ \frac{E(y) - R_0}{R_1 - R_0} \right]
\]  

(19)

and the residential health-quality variance is

\[
VAR(\theta) = \left( \frac{\alpha}{\alpha + \beta}(R_1 - R_0) \right)^2 VAR(y).
\]  

(20)

Compatibly with the statistical notion of the concentration coefficient, the residential
dispersion coefficient (RDC) is defined as the ratio of residential health-quality variance
to the residential health-quality mean

\[
RDC = \left( \frac{\alpha}{(\alpha + \beta)(R_1 - R_0)} \right) \frac{VAR(y)}{E(y) - R_0}
\]  

(21)

where \( VAR(y) / (E(y) - R_0) \) can be interpreted as the base-rent-adjusted income-
dispersion coefficient.

Corollary 10: When all people have identical health sensitivity to environmental health
quality and identical utility’s consumption elasticity, the residential dispersion coefficient
is proportional to the base-rent adjusted income-dispersion coefficient. (See Appendix D
for proof.)
Corollary 11: When all people have identical health sensitivity to environmental health quality and identical utility’s consumption elasticity, the effect of the base-rent adjusted income-dispersion coefficient on the residential dispersion coefficient is intensified by the population’s health sensitivity to environmental health quality and moderated by the population’s consumption-elasticity of utility and the market rent-gradient. (See Appendix D for proof.)

Let us now consider the case where people’s health sensitivities to environmental health quality and people’s consumption elasticities are not identical and let us denote the population means of health sensitivity to environmental health quality, utility’s consumption elasticity and income by $\mu_\alpha$, $\mu_\beta$ and $\mu_y$, respectively. Considering the first-order approximation of the interior solution displayed by Eq. (13) in the vicinity of these means, the variance of the chosen health quality of the place of residence within the population is given by:

$$VAR(\theta) \equiv \left(\frac{\mu_y - R_0}{(\mu_\alpha + \mu_\beta)(R_1 - R_0)}\right)^2 \{\mu_\beta^2 VAR(\alpha) + \mu_\alpha^2 VAR(\beta) + [\mu_\alpha (\mu_\alpha + \mu_\beta) / (\mu_y - R_0)]^2 VAR(y)$$

$$- \mu_\beta \mu_\alpha COV(\alpha, \beta) + [\mu_\alpha (\mu_\alpha + \mu_\beta) / (\mu_y - R_0)] [\mu_\beta COV(\alpha, y) - \mu_\alpha COV(\beta, y)]\}.$$ (22)

In this case, the complexity of the residential dispersion coefficient hinders insight. Hence, the following corollaries are focused on the residential variance.

Corollary 12: If $\mu_\alpha$ is positive (zero), $\frac{\partial VAR(\theta)}{\partial VAR(y)}$ is positive (zero) and increasing in $\mu_\alpha$ and decreasing with $\mu_\beta$ and $R_1 - R_0$. (See proof in Appendix D.)

This corollary implies that as long as some people’s health is sensitive to environmental health quality, the residential-health variance increases with the income-variance. The larger the group of such people and the higher their health sensitivity to environmental health quality, the more profound the effect of the income variance on the residential-health variance. This effect is moderated by the average utility’s consumption elasticity within the population and by the market rent-gradient.

Corollary 13: The residential-health variance increases with the variance of the health sensitivity to environmental health quality and with the variance of the consumption elasticity of utility within the population. (See proof in Appendix D.)

Corollary 14: The residential-health variance increases with the covariance between income and environmental sensitivity within the population, but decreases with the covariance between environmental sensitivity and consumption elasticity of utility and with the covariance between income and consumption elasticity of utility within the population. (See proof in Appendix D.)

Corollary 15: If the mean of the environmental sensitivity is larger (smaller) than the mean of the consumption elasticity of utility within the population, the moderating effect of the covariance between income and consumption elasticity of utility dominates (is
dominated by) the intensifying effect of the covariance between income and environmental sensitivity on the residential-health variance. (See proof in Appendix D.)

7. Conclusion

A lifetime utility function was constructed under the assumption that the duration of life is uncertain and depends upon the health quality of the place of residence and the length of the period of exposure to hazardous facilities and land use that exist in that place. Using this health-quality adjusted utility function, the effect of environmental-health quality on the highest bid on a residential property was found to be positive and intensified by income and environmental-health sensitivity, but moderated by the consumption elasticity of utility, rate of time preference and rate of decline of survival probability stemming from continued exposure to hazardous facilities and land use. It was shown that when income is not affected by environmental-health quality, the optimal health quality of the place of residence is proportional to the ratio of the difference between the individual income and the rent in the most hazardous location to the difference between the rents in the least and most hazardous locations. The proportion-coefficient rises with the individual’s sensitivity to environmental-health quality and declines with his utility’s consumption-elasticity.

The possibility that productivity is positively affected by environmental-health quality was considered. It was found that when an interior solution to the location choice problem exists, the optimal environmental-health quality of the place of residence increases with the personal sensitivity to the environmental-health quality, with the marginal effect of the environmental-health quality on income and with the income-rent differential in the most hazardous location, but decreases with the utility’s consumption elasticity and with the marginal effect of the environmental-health quality on rent. Under certain circumstances the solution to the location-choice problem is corner. When income in the most hazardous location is at least as large as the rent and when the environmental-health-quality effect on income is at least as large as its effect on rent, people maximize their consumption and minimize their risk of dying by residing in the least hazardous location. However, when income in the most hazardous location is equal to the rent and the environmental-health-quality effect on the market rent exceeds its effect on income people reside in the most hazardous location as they cannot afford renting, or bearing the user cost of, a property in a less hazardous location.

When people are endowed with identical health sensitivities to environmental health hazards and consumption elasticities of utility, their residential dispersion is closely related to the distribution of income. In the more likely case of heterogeneous population, the level of residential dispersion increases with the variances of income, environmental-health sensitivity and utility’s consumption elasticity and with the covariance between income and environmental-health sensitivity within the population, but decreases with the covariances between environmental health sensitivity and utility’s consumption elasticity and between income and utility’s consumption elasticity within the population. The moderating effect of the covariance between income and utility’s consumption elasticity on the level of residential dispersion dominates the intensifying effect of the covariance between income and environmental-health sensitivity on the level of residential dispersion when the mean of environmental-health sensitivity is larger than the mean of utility’s consumption elasticity within the population.
Frequently, political power is related to economic power, in which case a strong association between residential location and income might contribute to the persistence of hazardous facilities and land use in the vicinity of poor neighborhoods, whose lobbying effort is the least effective. The interior solution to the location-choice problem implies that a high level of income inequality within a population of rational people is a necessary, but not sufficient, condition for a strong association between residential location and income. A low correlation between income and sensitivity to environmental-health quality and a high correlation between income and utility’s consumption elasticity within the population may lead to residential dispersion where household income is not correlated with neighborhood’s environmental-health quality. However, the existence of hazardous facilities and land use in, or near, populated areas is perpetuated by a strong association between residential location and income, and, in turn, intensifies the association between residential location and income, when there is a high correlation between income and health sensitivity to environmental-health quality and a low correlation between income and utility’s consumption elasticity within the population. Furthermore, the higher the income-disparity level the greater the likelihood of persistent hazardous facilities and land use in the vicinity of poor neighborhoods. When the level of income disparity is very high, corner solutions to individuals’ location choice problems are likely to be a common phenomenon, leading to residential polarization of the population: low-income earners living in low environmental-health-quality locations and high-income earners residing in high environmental-health-quality neighborhoods.

Appendix A: An explanation of the transition from Eq. (1) to Eq. (2)

Recall that 
\[ f(t) = F'(t) \] 
where the subscript \( j \) is omitted for convenience. Then, Eq. (1) can be rendered as

\[
V = \int_0^\infty F'(t) \left\{ \int_0^t e^{-\rho \tau} u_\tau d \tau \right\} dt = \int_0^\infty v \, dU \tag{A2}
\]

where,

\[
v = \int_0^t e^{-\rho \tau} u_\tau d \tau \tag{A3}
\]

and

\[
U = -(1 - F(t)). \tag{A4}
\]

The integration by parts rule suggests that

\[
V = \int_0^\infty v \, dU = Uv - \int_0^\infty U \, dv. \tag{A5}
\]

Note, however, that

\[
Uv = - \left[ (1 - F(t)) \int_0^t e^{-\rho \tau} u_\tau d \tau \right]_0^\infty = 0 \tag{A6}
\]

because when evaluated at the lower limit
\[ Uv = -\left[ (1 - F(0)) \int_0^\infty e^{-\rho \tau} u_\tau d \tau \right] = 0 \]  
(A7)

and when evaluated at the upper limit

\[ Uv = -\left[ (1 - F(\infty)) \int_0^\infty e^{-\rho \tau} u_\tau d \tau \right] = 0 \]  
(A8)

as

\[ F(\infty) = 1. \]  
(A9)

Hence,

\[ V = -\int_0^\infty Ud\nu. \]  
(A10)

By virtue of equation (A3)

\[ dv = e^{-\rho \tau} d\tau \]  
(A11)

and the substitution of equations (A4) and (A11) into (A10) implies

\[ V = \int_0^\infty e^{-\rho \tau} u_\tau \Omega(t) dt \]  
(A12)

where

\[ \Omega(t) \equiv -U_\tau = 1 - F(t) \]  
(A13)

and indicating the probability of living at least until \( t \).

**Appendix B: Proofs of corollaries 1 and 2**

**Proof of Corollary 1:** For \( 0 < \alpha_j \leq 1 \) the fact that \( \theta_B^j / \theta_A^j < 1 \) implies that

\[ u_j(y_j - R_{A}^j) > u_j(y_j - R_{B}^j) \]  
and, recalling that \( u'_j > 0, \; R_{A}^j > R_{B}^j \). For \( \alpha_j = 0 \),

\[ u_j(y_j - R_{B}^j) = u_j(y_j - R_{A}^j) \]  
and, recalling \( u'_j > 0, \; R_{A}^j = R_{B}^j \).

**Proof of Corollary 2:** By differentiating Eq. (10),

\[ \frac{\partial(P_A^j / P_B^j)}{\partial(\theta_A / \theta_B)} = \frac{\alpha_j(y_j - R_A^j)}{\beta_j(\rho_j + \mu_j)}(\theta_A / \theta_B)^{-\alpha_j / \beta_j + 1} > 0 \]  
and increases with \( y_j \) and \( \alpha_j \) but decreases with \( \beta_j, \; \rho_j \) and \( \mu_j \).

**Appendix C: First and second order conditions for maximum and proofs of corollaries 3-7**

The location-choice problem is

\[ \max_{\hat{\theta}} \{ V = \theta^a c(c(\theta)) / (\rho + \mu) \} \]

where the index \( j \) is omitted for tractability and \( c \equiv y - R \).

The first-order condition (f.o.c.) for maximum

\[ \frac{dV_j}{d\theta} = \theta^{a-1} [ac(c(\theta)) + \theta u'(c(\theta))c'(\theta)] = 0 \]

implies

\[ \theta = \frac{ac(c(\theta))}{c'(\theta)u'(c(\theta))}. \]
The second-order condition (s.o.c.) for maximum is
\[
\frac{d^2V_j}{d\theta^2} = (\alpha - 1)\theta^{\alpha-2} \left(cu + \theta u'c'\right) + \theta^{\alpha-1} \{ (1 + \alpha)u'c' + \theta[u^*c'^2 + u'c'c'] \} < 0.
\]
From the f.o.c., the first term on the r.h.s. of the s.o.c. is equal to zero, and as \( c(\theta) \) is taken to be linear, the s.o.c. can be rendered as
\[
(1 + \alpha)u'(c(\theta)) < \theta u^*(c(\theta))c'(\theta).
\]
Recalling also that \( u = c^\beta \),
\[
(1 + \alpha)\beta c(\theta)^{\beta-1} < \beta c'(\theta)
\]
or, equivalently,
\[
(1 + \alpha)c(\theta) < \theta c'(\theta).
\]
By rearranging terms,
\[
\frac{c(\theta)}{c'(\theta)} > \frac{1}{\alpha + 1}.
\]
When only rent is affected by location,
\[
c(\theta) = y - [R_0 + (R_1 - R_0)\theta]
\]
and the s.o.c. for maximum requires
\[
\frac{y - R_0}{R_1 - R_0} > \frac{1}{\alpha + 1} \theta
\]
or, equivalently,
\[
\frac{y - R_0}{R_1 - R_0} > [1 + \frac{1}{\alpha + 1}]\theta
\]
where, by virtue of our assumption \( 0 \leq \alpha \leq 1 \) and \( 0 < \beta < 1 \), the term on the r.h.s. is positive.
When income is also affected by location
\[
c(\theta) = [y_0 + (y_1 - y_0)\theta] - [R_0 + (R_1 - R_0)\theta]
\]
the s.o.c. for maximum requires
\[
\frac{y_0 - R_0 + [(y_1 - y_0) - (R_1 - R_0)]\theta}{-[(y_1 - y_0) - (R_1 - R_0)]} > \frac{1}{\alpha + 1} \theta
\]
or, equivalently,
\[
\frac{y_0 - R_0}{(R_1 - R_0) - (y_1 - y_0)} > [1 + \frac{1}{\alpha + 1}]\theta
\]
where the term on the r.h.s. is positive.

**Proof of Corollary 3:** If \( \frac{y_j - R_0}{R_j - R_0} > [1 + \frac{1}{\alpha_j}]\theta_j^* \), there exists an interior solution to the location-choice problem. By substituting Eq. (12), \( R' = R_1 - R_0 \) and \( u = c^\theta \) into Eq. (11), \( \theta_j^* = \left( \frac{\alpha_j}{\alpha_j + \beta_j} \right) \left[ \frac{y_j - R_0}{R_1 - R_0} \right] \). The rest is straightforward from this expression.
Proof of Corollary 4: From Eq. (13), \( \theta_j^* = 1 \) if
\[
\left( \frac{\alpha_j}{\alpha_j + \beta_j} \right) \left[ \frac{y_j - R_0}{R_1 - R_0} \right] \geq 1.
\]
By rearranging the terms in this inequality, \( \theta_j^* = 1 \) if
\[
y_j \geq R_0 + (1 + \frac{\beta_j}{\alpha_j})(R_1 - R_0) = R_1 + (\frac{\beta_j}{\alpha_j})(R_1 - R_0).
\]

Proof of Corollary 5: The s.o.c. requires that
\[
\frac{(y_0 - R_0)}{(R_1 - R_0)} - (y_1 - y_0) > [1 + \frac{1 - \beta}{1 + \alpha}]\theta,
\]
where the term on the r.h.s. is positive. If \( y_0 - R_0 > 0 \) the satisfaction of the s.o.c. requires that \( R_1 - R_0 > y_1 - y_0 \). If \( y_0 - R_0 < 0 \) the satisfaction of the s.o.c. requires that \( R_1 - R_0 < y_1 - y_0 \).

Proof of Corollary 6: Straightforward from Eq. (16).

Proof of Corollary 7: \( \frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{1j} - y_{0j})} < 0 \) and hence the s.o.c. for maximum is not satisfied and the solution to the location-choice problem is corner: \( \theta_j^{**} = 0 \) or 1. When \( y_{0j} \geq R_0 \) and \( y_{1j} - y_{0j} \geq R_1 - R_0 \) environmentally sensitive individuals maximize their consumption and minimize their risk of dying by residing in the least hazardous environment. When \( y_{0j} = R_0 \) and \( y_{1j} - y_{0j} < R_1 - R_0 \) people reside in the most hazardous location since safer ones are not affordable.

Proof of Corollary 8: From Eq. (16), \( \theta_j^* = 1 \) if
\[
\left( \frac{\alpha_j}{\alpha_j + \beta_j} \right) \left[ \frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})} \right] \geq 1.
\]
By rearranging the terms in this inequality, \( \theta_j^* = 1 \) if
\[
y_{1j} \geq R_0 + (1 + \frac{\beta_j}{\alpha_j})[(R_1 - R_0) - (y_{1j} - y_{0j})] = R_1 + (\frac{\beta_j}{\alpha_j})[(R_1 - R_0) - (y_{1j} - y_{0j})]
\]

Appendix D: Proofs of corollaries 9 and 15

Proof of Corollary 9: Straightforward from Eq. (18).

Proof of Corollary 10: Straightforward from Eq. (21).

Proof of Corollary 11: Straightforward from Eq. (21).

Proof of Corollary 12: From Eq. (22),
\[
\frac{\partial \text{VAR}(\theta)}{\partial \text{VAR}(y)} = [\mu_\alpha/(\mu_\alpha + \mu_\beta)(R_1 - R_0)]^2 \geq 0
\]
as \( \mu_\alpha \geq 0 \), which further implies that
\[
\frac{\partial^2 \text{VAR}(\theta)}{\partial \text{VAR}(y)\partial \mu_\alpha} > 0 \quad \text{and} \quad \frac{\partial^2 \text{VAR}(\theta)}{\partial \text{VAR}(y)\partial \mu_\beta} < 0
\]
and
\[
\frac{\partial^2 \text{VAR}(\theta)}{\partial \text{VAR}(y)\partial (R_1 - R_0)} < 0.
\]

Proof of Corollary 13 and Corollary 14: Straightforward from Eq. (22) as long as \( \mu_y - R_0 > 0 \). Supply and demand consideration and the assumption that rent increases with environmental quality imply that \( \mu_y - R_0 > 0 \). Otherwise, people with income lower than the mean, as well as people earning the mean income and consume, could not afford renting a residential property.

Proof of Corollary 15: From Eq. (22),
\[ \frac{\partial \text{VAR}(\theta)}{\partial \text{COV}(\beta, y)} - \frac{\partial \text{VAR}(\theta)}{\partial \text{COV}(\alpha, y)} \geq \left( \frac{\mu_y - R_0}{(\mu_\alpha + \mu_\beta)^2 (R_1 - R_0)} \right)^2 (\mu_\alpha - \mu_\beta) = 0 \text{ as } (\mu_\alpha > \mu_\beta). \]

REFERENCES


